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Developing argumentation in mathematics: The role of evidence and context

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Abstract

Multiple potential benefits to the introduction of argumentation into classroom environments have been identified and documented, including the potential to extend teaching goals to emphasise cognitive and metacognitive processes, epistemic criteria and reasoning, as well as the enculturation of students into the practices and discourses of a subject. Argumentation structures and practices offer the means to focus students on the need for quality evidence, potentially encouraging students to focus deeply on mathematical content. Much of the work with argumentation that has already occurred in mathematics is associated with justification of procedural choices to arrive at a correct answer. By contrast, mathematical inquiry offers the opportunity for students to engage in ill-structured, ambiguous problems that have neither a defined solution path nor a single correct answer. Thus there is great potential for argumentation to be effective in inquiry-based learning environments. However, very little research has focused on argumentation practices of students undertaking inquiry of this nature.

The study presented is thus exploratory; designed to both develop deeper understanding of Inquiry-Based Argument practices and possibilities, and to identify how students' developing use of evidence in argumentation could be understood and supported. Specifically, the study sought to address the following research questions:

1. What are key features of an Inquiry-Based Argument model as implemented in a primary (elementary) mathematics setting?
2. What signature elements of Inquiry-Based Argument can serve to guide children's mathematical argumentation?

A design research methodology approach was utilised, with iterative cycles of inquiry and argumentation implemented in a single inquiry classroom of Year 4-5 students (n=27, aged 8-10). These cycles introduced the role of evidence, the structure of argument, and evidence and argument quality, to a class environment that embraced a knowledge building culture (Bereiter & Scardamalia, 1996).

Berland and Reiser's (2009) Goals of Argument and McNeill and Martin's (2011) Conclusion-Evidence-Reasoning Framework were used to guide the instructional approach which was modified progressively to meet the developing needs of the students.

Data were generated through videos of emerging classroom practices, interviews with students, student work artefacts, teaching notes, and observations over the course of ten months. A Grounded Theory Approach (Corbin & Strauss, 2008) was utilised for data analysis. The analysis identified several significant results of introducing Inquiry-Based Argument into the classroom:

1. Introducing argument practices enabled the ‘visibilising’ of student thinking, increasing opportunities to develop and utilise cognitive conflict.
2. An increased focus by students on the need for quality evidence enables them to be able to put forth and negotiate arguments.

In turn, these factors appear to enable students to:

1. Develop complex appreciation for argument structure, progressing from intuitive responses to development of qualified arguments.
2. Demonstrate understanding and appreciation of the critical role of evidence and evidence quality in the development of deepened mathematical understandings and argumentation processes and structure.

Particular areas that presented difficulty for the students were observed throughout the unit and teaching and learning supports were noted as these offer potential for scaffolding argumentation development.

Two tangible contributions from the research include a proposed model of inquiry-based mathematical argumentation, based on integration between Mathematical Knowledge, Argumentation Knowledge and Context Knowledge, and a detailed guideline for measurement of developing argumentation practices with young students in this context.

While this research is limited in that it followed one class of students only, restraining the size of the research enabled a deeper analysis of the experiences of these 27 students. This research suggests that there is potential for argumentation to have a significant role in mathematics education and that it is certainly worthy of further research.

Declaration by author

This thesis is composed of my original work, and contains no material previously published or written by another person except where due reference has been made in the text. I have clearly stated the contribution by others to jointly-authored works that I have included in my thesis.

I have clearly stated the contribution of others to my thesis as a whole, including statistical assistance, survey design, data analysis, significant technical procedures, professional editorial advice, and any other original research work used or reported in my thesis. The content of my thesis is the result of work I have carried out since the commencement of my research higher degree candidature and does not include a substantial part of work that has been submitted to qualify for the award of any other degree or diploma in any university or other tertiary institution. I have clearly stated which parts of my thesis, if any, have been submitted to qualify for another award.

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Publications during candidature

Journal Articles

Fielding-Wells, J., Dole, S. & Makar, K. (2014). Inquiry pedagogy to promote emerging proportional reasoning in primary students. *Mathematics Education Research Journal, 26*(1), 1-31.

Makar, K. & Fielding-Wells, J. (submitted). Concept-driven disequilibrium: supporting emerging statistical concepts in complex problems. *Educational Studies in Mathematics*.

Peer Reviewed Conference Proceedings

Fielding-Wells, J. (2010). Linking problems, conclusions and evidence: Primary students' early experiences of planning statistical investigations. In C. Reading (Ed.), *Proceedings of the 8th International Conference on Teaching Statistics. Voorburg, The Netherlands: International Statistical Institute.*

Fielding-Wells, J., & Makar, K. (2012). Developing primary students' argumentation skills in inquiry-based mathematics classrooms. In K. T. Jan van Aalst, Michael J. Jacobson and Peter Reimann (Ed.), *The future of learning: Proceedings of the 10th International Conference of the Learning Sciences [ICLS 2012] - Volume 2 short papers, symposia, and abstracts.* (pp. 149-153). Sydney, NSW, Australia: International Society of the Learning Sciences.

Fielding-Wells, J. (2013). Inquiry-Based Argumentation in Primary Mathematics: Reflecting on Evidence. *Thirty-sixth Mathematics Education Research Group of Australasia Conference [MERGA 36].* Melbourne, Australia, 7-11 July 2013.

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Peer Reviewed Conference Papers

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Makar, K., Fielding-Wells, J. & Allmond, S. (2011). *Is This Game 1 or Game 2? Primary Children's Reasoning about Samples during Inquiry.* Paper presented at the Seventh International Research Forum on Statistical Reasoning, Thinking and Literacy (SRTL7), Texel Island, The Netherlands.

Book Chapter

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Keywords

argumentation, mathematics education, evidence, context, Inquiry-Based Argument, primary education

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List of Abbreviations used in the thesis

ACARA	Australian Curriculum Assessment and Reporting Authority
CER	Claim-Evidence-Reasoning Model
IBA	Inquiry-Based Argument
IBL	Inquiry-Based Learning
NAPLAN	National Assessment Program: Literacy and Numeracy
NCTM	National Council of Teachers of Mathematics
NNRP	National Numeracy Review Panel
NRC	National Research Council
OECD	Organisation for Economic Co-operation and Development
PISA	Programme for International Student Assessment
TIMSS	Trends in International Mathematics and Science Study

1 Introduction

1.1 Chapter Overview

Societal, political, economic, and technological change has brought about new demands within the workforce. As one of the purposes of education is ostensibly to prepare students to become productive citizens, it follows that education needs to respond to workforce requirements. One area in which this does not appear to be happening is mathematics education, where sufficient demands for an appropriately skilled workforce are not likely to be met in the coming years. Research suggests that this largely is a product of students feeling that mathematics is not particularly useful or interesting, and that it is a difficult subject to undertake and succeed in (McPhan, Moroney, Pegg, Cooksey, & Lynch, 2008). Calls have also been made for students to engage in mathematics in more authentic ways, as practitioners and experts would (Brown, Collins, & Duguid,1989). This push for reform has not been widely adopted with predominantly traditional methods of teaching still widespread. Criticisms of reform have also come from several quarters, with one dominant opposition being the potential for the loss of mathematical integrity in the teaching: the ‘dumbing down’ or ‘watering down’ of the mathematics content.

Argumentation practices have been used in science learning for some time as a means of inculcating students into the discipline authentically and thus may offer potential in mathematics teaching and learning. Argumentation practices rely on the obtaining of evidence, using evidence to make a claim, and then articulation of how the evidence leads to the claim through reasoning. As such, argumentation offers potential to purposefully direct students to focus on the discipline content, and the ways in which the discipline content can be used, to respond to a problem or dilemma. However, there are significant differences between science and mathematics as learning areas: mathematics has potential to be taught at a predominantly abstract level. In order to implement argumentation with children, mathematics ideally needs to be embedded in a context that enables argument to take place. The ill-structured, ambiguous nature of inquiry-based learning offers a means for embedding argumentation into context-rich learning environments.

This introductory chapter serves to set the scene for the research by providing some background into the Australian context, and defining some of the key concepts relevant to their use in this research.

1.2 Background Context

Twenty-first century society demands a workforce that can communicate effectively, problem-solve, make decisions based on critical thinking, and understand complex systems (Carew, 2004). Of course, the ability to think critically and analytically are not new skills, but rather have been identified as increasingly necessary in the last few decades as the nature of work and the economy have changed (Silva, 2009). Silva argues that *“today’s workers in nearly all sectors of the economy must be able to find and analyse information, often coming from multiple sources, and use this information to make decisions and create new ideas” (2009, p.630).*

These societal and technological changes have not gone unnoticed. Educational policy in recent years has made a pronounced shift towards the development of a workforce that can participate in the new societies of the 21st century, and position Australia favourably as a nation. A necessary shift, as “the competitive strength of a nation in the modern world is directly proportional to its learning capacity; that is, a combination of the learning capacities of the individuals and the institutions of the society.” (Papert, 1993, p. vii). This view is reflected in the intent of the Australian curriculum shaping document which states that the Australian Curriculum:

> has been written to equip young Australians with the knowledge, understanding and skills that will enable them to engage effectively with, and prosper in, society, to compete in a globalised world and to thrive in the information-rich workplaces of the future. (ACARA, 2012, p.28)

This intent raises significant questions specifically for mathematics education, as the mathematical sciences are fundamental to the well-being of all nations: driving the data analysis, forecasting, modelling, decision-making, management, design, and technological principles that are the foundation of every sector of modern enterprise. Mathematics is the foremost enabling science which underpins research, development, and innovation in every aspect of society, from business and science through to health and national security

(Australian Academy of Science, 2006, p. 18). The need for a mathematically capable workforce gives rise to a number of concerns: whether Australia has the numbers of students engaged in higher level mathematics to meet demand; whether Australian students are capable of meeting demand; and, whether we are using pedagogical practices that contribute effectively towards this goal. Some of the issues surrounding these questions will be addressed briefly in this introduction in order to provide a backdrop to the research reported in this document, and to explicate the necessity of this research at a point in Australian mathematics education that has already extended well beyond critical.

The importance of a supply of capable, mathematically-trained citizens, in an increasingly technological society cannot be overemphasised; yet international trends indicate that, while the demand for Science, Technology, Engineering and Mathematics (STEM) skills is increasing, the level of student participation in mathematics is steadily declining in many countries (OECD, 2006). This is also the case in Australia; as was highlighted in the *National Strategic Review of Mathematical Research in Australia* (Australian Academy of Science, 2006) and *Maths! Why Not?* (McPhan, Moroney, Pegg, Cooksey, & Lynch, 2008), which came with a warning that Australia is unlikely to produce enough tertiary graduates from mathematical disciplines to meet workforce requirements. Essentially, “Australia will be unable to produce the next generation of students with an understanding of fundamental mathematical concepts, problem-solving abilities and training in modern developments to meet projected needs and remain globally competitive.” (Australian Academy of Science, 2006, p.9). The problem does not begin at the tertiary level: since 1995, the Australian Mathematical Sciences Institute (AMSI) has been reporting the numbers of Australian students engaged in mathematics at the senior secondary level (the last two years of formal schooling and prior to university entrance). These figures indicate that from 1995 to the most recently available numbers in 2012, Australia has experienced steadily declining proportions of students engaged in Advanced and Intermediate levels of mathematics (14.1% to 9.4% and 27.2% to 19.4% respectively) with a corresponding increase in the numbers of students taking elementary mathematics subjects (37% to 52%) (Barrington, 2011, 2012, 2013). This leads to the question of whether there are significant flaws in the Australian education system that fail to prepare students adequately to engage in higher mathematical learning.

In the context of tracking school mathematical knowledge, the two international studies of the greatest breadth and depth are the PISA study (Performance Indicators of Student Achievement) and TIMSS (Trends in International Mathematics and Science Study). Each of these studies provides detailed insights into Australian mathematics education trends.

The most recent PISA study, in 2012, identified Australia as one of the highest PISA performers among OECD countries. Australia's education system was described as having

> fair and inclusive practices that strive for equity, quality and high completion rates for upper secondary and tertiary education. Australia has fewer underperforming students than the OECD average, a high proportion of children enrolled in early childhood education, and comprehensive school until age 16. … Australia's schools have positive learning environments, strong pedagogical leadership and well-prepared teachers, all supported with an effective evaluation and assessment framework. Students' instruction times and teachers' teaching time are among the highest across OECD countries. (OECD, 2013, p. 4)

This initially gives the impression that Australia has an enviable schooling system, and would be supported by Australia's high placement with regards to other countries in the last round of testing. However, this doesn't paint the whole picture. The average mathematical literacy performance of Australia has declined consistently and significantly (by 20 score points on average) between PISA 2003 and PISA 2012, with no significant corresponding change in the OECD average over this time (Thomson, De Bortoli, & Buckley, 2013, pp. xiii-xiv). These summary findings indicate Australia to be one of the five countries to have experienced the greatest overall decline over the decade. This appears to be partially a result of shifts in both tails of the results, with a significant decline (5%) in the proportion of Australian students reaching the highest level of proficiency and a significant increase (5%) in the proportion of low performers (Thomson et al., 2013). The PISA results also illustrate a concerning educational disadvantage attached to low socio-economic background, where the performance gap between students of the same age from different backgrounds can be equivalent to up to three years of schooling. Thomson, De Bortoli, Nicholas, Hillman, and Buckley (2011, p. xiv) argue that "this gap places an unacceptable proportion of 15-year-old students at serious risk of not achieving levels sufficient for them to effectively participate in the 21st century workforce and to contribute to Australia as productive citizens."

Essentially the PISA data tells us that Australia has a strong schooling system, with positive learning environments and high rates of completion. Furthermore, Australian students spend significant amounts of time engaged in mathematics instruction: more than the international average (Thomson et al., 2012b, 2012c). However, time spent does not guarantee learning and consideration must be given to the activities that are undertaken during that time, and TIMSS provides some insight. In the 2011 study of Year 4 and Year 8 students, Australian students reported spending less time than the international average working on: *problems, individually or with peers, with teacher guidance* (reported at the Year 4 level – although at Year 8 this is higher than international average); *problems together in whole class with direct teacher guidance;* and, *explaining answers.* Conversely, more time was spent *working on problems (individually or with peers) while teacher is doing other tasks* (Thomson et al., 2012a). This suggests students work a significant amount of the time independently, with less opportunity to explain and reason their understandings and problem responses with teachers, and that even at the Year 8 level, the teacher guidance is largely given at the individual or small group level, rather than whole class discussion of problems.

The TIMSS International Study provides Benchmark descriptors (Mullis, Martin, Foy, & Arora, 2012, pp. 87, 113) for each of the year levels (Table 1.1). In 2011, 35% of Year 4 students reached the High Benchmark or above, while 29% of Year 8 students reached this level. This suggests that only around a third of Australian students are skilled at applying knowledge and understanding to solving problems (Year 4) or applying understanding and knowledge in complex situations (Year 8). The same results show, 10% of Year 4 students reached the Advanced Benchmark, while 9% of Year 8 students reached this level. This further suggests that only around a tenth of Australian students are skilled at explaining their reasoning (Year 4) or reasoning, concluding and making generalisations (Mullis et al., 2012, pp. 90, 114).

If such is the case, then surely an improvement agenda needs to be focussed towards engaging Australian students in opportunities to apply their knowledge to problems and to have experiences in explaining and reasoning through their conclusions. Such an agenda needs to be addressed soon if Australia is to meet its stated goal of having, by 2025, “the performance of our students in STEM disciplines rank[ing] among the best of their international peers” (Office of the Chief Scientist, 2013, p. 7): especially given that the PISA studies warn us that the quality of mathematics learning previously experienced in Australia is slipping relative to other countries.

Table 1.1: TIMSS 2011 International Benchmarks of Mathematics Achievement (Mullis et al., 2012, pp. 87, 113)

Advanced International Benchmark [Scale Score 625]	
Year 4	Students can apply their understanding and knowledge in a variety of relatively complex situations and explain their reasoning.
Year 8	Students can reason with information, draw conclusions, make generalizations, and solve linear equations.
High International Benchmark [Scale Score 550]	
Year 4	Students can apply their knowledge and understanding to solve problems.
Year 8	Students can apply their understanding and knowledge in a variety of relatively complex situations
Intermediate International Benchmark [Scale Score 475]	
Year 4	Students can apply basic mathematical knowledge in straightforward situations.
Year 8	Students can apply basic mathematical knowledge in a variety of situations.
Low International Benchmark [Scale Score 400]	
Year 4	Students have some basic mathematical knowledge.
Year 8	Students have some knowledge of whole numbers and decimals, operations, and basic graphs.

What should also be of concern for Australia is that, given a relatively small population governed by a small number of educational overseeing bodies, the country could and should have responded more quickly to calls for reform made over two decades ago.

In 1991, the US Professional Standards for Teaching Mathematics (NCTM, 1991) were published, which stated that mathematics classrooms must move towards:

- Becoming mathematical communities rather than collections of individuals;

- Use of logic and mathematical evidence for verification rather than the teacher being the sole authority;
- A focus on reasoning rather than memorisation;
- Conjecturing, inventing, and problem solving being used rather than mechanistic answer-finding; and
- Connections being made between mathematics, its ideas, and its applications rather than mathematics being seen as a body of isolated concepts and procedures.

Almost a decade later, a second significant emphasis came from the *Principles and Standards for School Mathematics* (NCTM, 2000) which explicitly built problem solving, and reasoning and proof, into its curricular vision. In 2008, the Australian National Numeracy Review Report (National Numeracy Review Panel [NNRP], 2008) came about in response to a need for improving numeracy and mathematics learning competencies within Australia (p. vii). This document was underpinned by research-based evidence sourced both domestically and internationally. The NNRP sought to address this research in light of Australian student performance in both national benchmarks and international testing results, such as PISA and TIMSS. The outcome of this report was a series of recommendations, directed specifically towards the Australian context, and yet showing a similar focus to the NCTM documents. Recommendations were made for improving numeracy outcomes, of which the two most pertinent to this research are provided below:

> 1. That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom …
> 2. That from the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking. (2008, p.xii)

In Australia, this focus is to an extent reflected in the intent of the K - 10 Curriculum for Mathematics (ACARA, 2014a) which identifies problem solving and reasoning as essential proficiencies. However, the format of the curriculum still privileges content knowledge over the proficiencies (Atweh, Miller, & Thornton, 2012).

1.3 **Addressing the problem**

If these recommendations are to be followed, if we are to see significant reform in Australian mathematics classrooms, the mere inclusion of ‘problem solving’ and ‘reasoning’ as isolated paragraphs in the Australian curriculum will not suffice (ACARA, 2014a). Rather, significant change in pedagogical approaches will need to occur in Australian classrooms: to rely on mathematical evidence for verification; to focus on reasoning; and, to conjecture and problem-solve. One potential pedagogical approach that may address this, argumentation, has experienced fairly extensive research interest in the field of science education.

Science and mathematics have historically experienced very similar approaches in terms of methodology and teaching pedagogy. Both are often seen as subjects involving a linear progression of knowledge to be established successively in order to develop deeper understandings. A heavy reliance on prescribed rules and formulas, that can be empirically supported and ‘proven’, makes up the body of mathematics or science. Such a

> “positivist” view of science, placing as it does emphasis on factual recall with confirmatory experiments, denies the role of the historical and social accounts of science, presenting science as a linear succession of successful discoveries. Applications of science, and their social implications, are simply limited to illustrations of the “use” to which scientific knowledge can be put. (Driver, Newton, & Osborne, 2000, p. 289)

However, in Australia, the science curriculum has increasingly taken an explicitly inquiry-based focus (ACARA, 2014b), intended to develop in students both the content needed for scientific understanding and the understanding and practice of scientific methodology to provide students the experience of ‘being’ a scientist. Internationally, science education has, in the last few decades, begun to emphasise the role of social interaction in learning and thinking processes and purports that higher thinking processes originate from socially mediated activities, particularly through the mediation of language (Jimenez-Aleixandre & Erduran, 2007). As a move to a socially mediated discursive approach, there has been a growing emphasis on the roles of argumentation in science education (Driver et al., 2000; Duschl & Osborne, 2002; S. Simon & Richardson, 2009) and it has now been fairly well established that “understanding the norms of scientific argumentation can lead students to understand the epistemological bases of scientific practice" (S. Simon & Richardson, 2009, p. 470). Argumentation practices are becoming increasingly researched and adopted in school science to reflect the actual work of the scientist, the building of scientific knowledge, and to address ethical arguments (e.g. stem cell research, and genetic selection and modification).

While argumentation practices have been researched widely in science education, this does not necessitate that it will work in mathematics education, despite the parallels that exist between science and mathematics in practice. One stumbling block may be the role of context, as scientific knowledge is highly contextualised while mathematical knowledge is often abstract and decontextualized. However, if argumentation as a pedagogical practice has demonstrated potential for deepening discipline-specific understandings in science education, a 'sister' science with mathematics, it is a worthwhile endeavour to consider its potential for similar affordances in mathematics education.

1.4 **Argumentation**

So what is meant by argumentation? In terms of language usage, a distinction can be made between the instrumental and argumentative uses of language. Instrumental uses are those which achieve a purpose directly without the need for reasoning or support. These might include greetings, a request for directions or a military command for example. They may achieve what was intended or may not; however they do not give rise to reasoning (Toulmin, Rieke, & Janik, 1984). Argumentative uses of language by contrast are those that either achieve their purpose or fail "only to the extent that they are 'supported' by arguments, reasons, evidence or the like …and have ...a rational foundation" (Toulmin et al., 1984, p. 5).

In everyday lay terms, arguments are often considered to occur in a somewhat confrontational manner whereby each side makes claims, defends them and argues against the opposing claims until a "winning" position has been established. By default then, there must also be a losing position. A classic example is that of the traditional debate, where both sides are able to take turns to put forward their theses, argue the opposing position and defend their own until a winner is declared. However, argumentation in the scientific sense is quite different, and the difference is identifiable by the aim or goal of the argument. Rather than aiming to achieve a "winning" position, argumentation in the scientific sense involves collaborative discussion to explore and resolve an issue in order to construct an explanation which best fits available evidence and logic (Berland & Reiser, 2009). For the purposes of the research presented here, argumentation will be used to refer to the whole activity of making claims, challenging them, backing them up by producing evidence and reasoning, criticising those reasons, rebutting those criticisms, and so on. The term reasoning will be used to describe the justification of evidence in support of a claim, so as to show how evidence justifiably leads to the claim. Argumentation is a complex, multifaceted construct; a form of discourse that needs to be appropriated by students and explicitly taught through instruction, task structuring and modelling (Jimenez-Aleixandre & Erduran, 2007, p. 4). In order to develop programs which can successfully support and scaffold students in this form of discourse, significantly more needs to be understood. This is not to say there has been no prior research in this area; to the contrary, a growing number of studies are focusing on the analysis of argumentation discourse in science learning contexts (see for example, Chin & Osborne, 2010; Driver et al., 2000; Jiménez-Aleixandre, Bugallo Rodríguez, & Duschl, 2000; Zohar & Nemet, 2002). There has also been prior research into argumentation practices in mathematics. However, this research has focused largely on mathematical proof (see, for example, Conner, 2007; Lampert, 1990). Krummheuer (1995, p. 235) cautions that

> If one uses the concept of argumentation in the field of mathematics, one might tend to bind it closely to that of proof. The analysis of argumentation in a classroom then, could be misleadingly understood as a treatise on proof. Therefore, one should notice that both the concept of argument and that of argumentation need not be exclusively connected with formal logic as we know it from such proofs or as the subject matter of logic.

Another area of argumentation research in mathematics has been that of argumentation as it applies to procedure (see, for example, R. Brown, 2007; Dixon, Egendoerfer, & Clements, 2009; Forman, Larreamendy-Joerns, Stein, & Brown, 1998; Goos, 2004; Krummheuer, 1995; Yackel & Cobb, 1996). Lampert asserts that it is “the strategies used for figuring out, rather than the answers, that are the site of the mathematical argument” (1990, p. 40). However, there is a further alternative, one that would appear to have been the focus of far less, if any, research and that is the use of argumentation to address authentic, ill-defined mathematical problems in which neither the procedural pathways nor the answers are limited in terms of ‘correctness’; that is, what Anderson describes as ‘inquiry’ (2002).

Blair (2012) describes a view of argumentation that essentially sees it as a form of inquiry in which argumentation is utilised to explore a problem and to arrive at a solution through examination of the evidence and grounds that can be employed towards a problem. It is this view of argumentation that is to be adopted as the theoretical basis of the research described here. For the research purpose of this paper, argumentation should be considered as a means to develop and test solutions to problems the students are posing, or being posed with, and to assess the effectiveness of those solutions against epistemic criteria. Thus, the research described in this paper differs from the existing body of literature somewhat in that both the solution process and the answers are considered the site of the argument. Hence, the term ‘Inquiry-Based Argumentation’ has been adopted.

1.5 Present Research

The aim of the research described in this dissertation was to develop pedagogical theory around Inquiry-Based Argumentation (IBA) in primary mathematics through multiple iterations of reflective-prospective cycles of improvement. While the research focussed specifically on argumentation, it was situated within a classroom which practiced inquiry-based learning (IBL) in order to have an established basis in which to situate an argument framework. The teacher involved firstly established an evidence-based approach with the classes, encouraging students to focus on the provision of quality evidence. Second, this was deepened into a model which incorporated reasoning as students progressed.

The results reported here were taken from a single class of students as they completed two units of work approximately ten months apart. Analysis of classroom discussion, student artefacts, and researcher reflections were used to develop a picture of classroom practices and outcomes. A tentative model of inquiry-based argument in mathematics is proposed which incorporates Context Knowledge, Argumentation Knowledge, and Mathematical Knowledge, along with the identification of some of the signature elements observed. Finally, suggestions for implications and further research are provided.

1.6 Summary

The intent of this chapter was to provide some background on the current Australian context in terms of current practice, and the mismatch between the curriculum as it is enacted, and curriculum intent as based on reform recommendations. These reforms address the need for pedagogies to be learner-centred and knowledge-based. One potential means of addressing reform is through inquiry-based learning. The research proposed here suggests a marriage of IBL with argumentation as a means of focussing students on the need to provide mathematically supported arguments when addressing ill-structured problems. Little research has been reported in the implementation of Inquiry-Based Argument as it is described here.

1.7 Dissertation Overview

Chapter 2: Philosophical Stance - will situate this research by addressing the philosophical underpinnings in order to make the positioning of the researcher apparent. The researcher's beliefs about scientific knowledge and learning, the nature of learners, and the nature of learning environments will be presented. This will be followed by consideration of the implications of such beliefs for the research.

Chapter 3: Literature Review - sets the scene for the research conducted by providing background literature on argumentation and argumentation practices. This chapter also addresses additional considerations instrumental to developing classroom argumentation practices including task considerations and student development.

Chapter 4: Theoretical Framework - enables the setting up and justification of the theoretical underpinnings of the models adopted for this study. Argumentation is a complex construct and this chapter addresses both structural and functional approaches to argument as well as potential for assessing argument quality.

Chapter 5: Methodology - explains and justifies the Design-Based approach taken and provides the reader with the background context and overview of the interventions implemented. Data collection, coding, and analysis methods are provided along with an overview of the development of a criteria sheet for assessing argument quality in the mathematics classroom.

Chapters 6 & 7: Results - sequentially presents the results of the two interventions addressed here. Each chapter reflects on the children's argumentation development both as a product (the argument itself) and as a process (the presentation of the argument): describing and analysing salient moments. Assessment of the students' argument development, both across each unit and as a whole, are also provided to illustrate growth of understanding of key elements of the practice.

Chapter 8: Discussion - presents the outcomes of the research: A model of Inquiry-Based Argumentation that incorporates three knowledge bases: mathematics, context and argumentation, and proposes potential interactions between these bases. Signature Elements, those elements that are characteristically essential to Inquiry-Based Argument in mathematics learning, are also proposed.

Chapter 9: Conclusion - As the purpose of this research was predominantly theory generation, the conclusion addresses the potential pragmatic implications for this research in the broader education settings. The reader is alerted to the limitations of this study and further research areas are suggested.

2 Philosophical Stance

2.1 Chapter Overview

In this section, the philosophical stance underpinning this research will be addressed in terms of the implications for research methodology, data collection, analysis and interpretation. The philosophical approach taken to research must essentially be expressed in order to identify the researcher's position and any influences brought to the research; including assumptions and beliefs about the nature of learning, choices made about methodology, approaches to data analysis and interpretation, and conclusions drawn from the data.

Beliefs about knowledge and the nature of knowledge impact significantly on the work of the researcher and serve to situate every aspect of research. "The relation between theory and practice is reflexive. Theory is seen to grow out of practice and to feedback and to inform and guide practice." (Cobb & Yackel, 1996, pp. 175-176). For example, the logical positivist might consider knowledge a search for 'truth': for that which can be scientifically verified or supported by mathematical proof, operating on the "premise that something is meaningful if and only if it is verifiable empirically" (Kincheloe & Tobin, 2009, p. 516). However, if a Radical Constructivist approach was to be adopted, then 'truth' would be argued as being cognitively constructed by the learner as experienced by that individual (Marshall, 1996; von Glasersfeld, 1989).

The work presented here draws essentially on the Knowledge Building framework conceptualised by Marlene Scardamalia and Carl Bereiter. Knowledge Building is situated more broadly within a constructivist paradigm and thus this chapter will briefly address that relationship before discussing the finer details of the approach. The remainder of the chapter has been divided into three progressive sections: beliefs about knowledge and learning, beliefs about learning environments, and the nature of role and context in learning.

2.2 Constructivism

Constructivism is essentially a psychological theory of knowledge underscored by two main principles: "(1) knowledge is not passively received but actively built up by the cognizing subject, [and] (2) the function of cognition is adaptive and serves the organization of the experiential world, not the discovery of ontological reality” (von Glasersfeld, 1989, p. 162). Not all constructivists accept both of these premises. von Glasersfeld coined the term Trivial Constructivism to refer to the acceptance of the first principle, and Radical Constructivism to refer to those who adopt both (Marshall, 1996). The research reported here has been designed under the broad principles of Radical Constructivism.

Reforms that have been promoted throughout education in the last few decades, such as those published by the NCTM (1991), are predominantly based on constructivist theories and positioning. Prior to constructivist-based reforms, a form of transmission-based instruction was more prevalent. In transmission-based instruction, the teacher is seen as the bearer or holder of the knowledge required to be transmitted to students. That knowledge is largely factual and procedurally based and its purpose is to be memorised and applied by students. Students are considered knowledgeable when they have collected a sufficient quantity of ‘knowledge’ and can demonstrate such through formalised assessment.

Confrey (1990, p. 107) describes three assumptions about direct teaching that constructivist approaches would challenge:

1. Relatively short products are expected from students, rather than process oriented answers to questions; homework assignments and test items are accepted as providing adequate assessment of the success of instruction.
2. Teachers, for the most part, can simply execute their plans and routines, checking frequently to see if the students’ responses are within desirable bounds, and only revising instruction when those bounds are exceeded (Petersen & Clark, 1978; Snow, 1972).
3. The responsibility for determining if an adequate level of understanding has been reached lies primarily with the teacher.

Transmission approaches can be considered teacher-centric. In contrast, a student-centred approach is developed largely from constructivist views, new technologies, and research on both content knowledge and classroom culture (NCTM, 2000; M. A. Simon, 1994). Clements and Battista (1990) summarise the basic tenets which underlie most constructivist theories as they apply to the educational setting:

1. Knowledge is actively created or invented by the child, not passively received from the environment.
2. Children create new mathematical knowledge by reflecting on their physical and mental actions. Ideas are made meaningful when they are integrated into children's existing structures of knowledge.
3. Individual interpretations are shaped by experience and social interactions. There is no one true reality. Learning mathematics is a process of adapting new information to existing understandings. Accordingly, students learn effectively during conflict and confusion (Confrey, 1985) as the disequilibrium necessitates a shift in understandings.
4. Learning is a social process: mathematical ideas and truths are cooperatively established by members of a culture. The constructivist culture sees students involved in social discourse involving explanation, negotiation, sharing and evaluation.

A comparison of the tradition model of learning with instruction based fundamentally on constructivist theory is provided in the Table 2.1 below.

Table 2.1 Traditional Mathematics versus Reform Mathematics (M. A. Simon, 1994, p. 79).

Traditional Instruction	Constructivist-Based Framework
No situations are available for students to communicate mathematical ideas and engage in negotiation of meaning.	Situations are established for students to communicate mathematical ideas and engage in negotiation of meaning.
Abstract ideas are presented followed by application in specific contexts.	Problem solving in presented in a specific context followed by abstraction/ generalisation of ideas.
The concepts discussed are presented by the teacher in his/her own language and /or the language of the communities in which he/she belongs.	The concepts discussed are developed by students and expressed in their language.
The responsibility for determining the validity of ideas rests with the teacher or is ascribed to the textbook.	The responsibility for determining the validity of ideas resides with the classroom community.
Application is limited to the practice and use of the general idea presented.	Application is the exploration of new ideas or extension of ideas previously developed.

However, it is not enough sufficient to say that learning takes place under a constructivist framework as constructivist learning spans a broad spectrum of classroom models and practices. Bereiter and Scardamalia (1996) identify five varieties of constructivist approach according to the extent to which they offer opportunities for a knowledge building focus:

1. Messing around (Discovery Learning): Students are provided with equipment, tools, materials and so forth and are expected to explore these. This may or may not result in significant outcomes; however, Bereiter and Scardamalia suggest “there is likely to be no objectification of knowledge … except possibly by the teacher” (1996, p. 502).

2. Hands-on Learning (Guided Discovery): While guidance is provided to the students, it is often with the aim of ‘discovering’ a known principle. Once the principle is ‘discovered’ there may be a tendency to stop the learning activity. A tendency to focus on physical objects, mental processes, and content is prevalent as distinct from the development of knowledge.

3. Learning through problem solving: Students engage collaboratively with a problem, but the actual aim is not knowledge building but rather using the problem as a tool to engage with the content of the learning.

4. Curiosity driven inquiry: The students address problems that stem from their own curiosity and gather information (empirical, observational, literature-based for example) to address the question. Bereiter and Scardamalia propose that this is knowledge building if the focus is on creating theories, explanations, hypotheses, and so on, but that attention needs to be given to improving upon these across the course of the inquiry.

5. Theory improvement: Students once again address problems that stem from their own questions but that there is an immediate shift by the teacher to encouraging initial conjectures by the students. The focus of class activities becomes improvement of these conjectures and justification of why a conjecture is an improvement over the previous one.

It is this last variety that incorporates Knowledge Building approaches. In order to appreciate the underlying basis for Knowledge Building it is necessary to first address the issue of what is meant by 'knowledge'. The following sections will address aspects of knowledge and learning; learners; and learning environments, from a broadly Knowledge Building perspective as applied to learners of science.

2.3 Nature of Scientific Knowledge and Learning

It is important to briefly consider what knowledge means in terms of scientific learning and within the scientific community. There is something of a tendency to equate knowledge in science with 'fact' and 'truth': an expectation that the scientific process sufficiently and rigorously enables the discovery of truth and enables truth to be identified. Whereas "even in relatively simple domains of science, the concepts used to describe and model the domain … are constructs that have been invented and imposed … often as results of considerable intellectual struggle." (Driver, Asoko, Leach, Mortimer, & Scott, 1994, p. 6). Thus, knowledge in scientific endeavour is rather a best explanation, a best understanding given the information available, and one which may be open to challenge as new information becomes available. Karl Popper (1972b) argued that the aim of science is not truth but verisimilitude:

> …while we *cannot* ever have sufficiently good arguments in the empirical sciences for claiming that we have actually reached the truth, we *can* have strong and reasonably good arguments for claiming that we may have made progress towards the truth; that is, that the theory T_2 is preferable to its predecessor T_1, at least in the light of all known rational arguments. (pp. 57-58)

Popper expands this to state that "one can never actually know whether one is getting closer to the truth" (1972a, p. 229). This is a logical assertion; if you know you are getting closer to the truth, then by implication you must know what the truth is, which means you have already arrived at the truth, so how do you get closer? (Bereiter, Scardamalia, Cassells, & Hewitt, 1997). If you cannot know if you are getting closer to the truth, then scientific progress must necessarily be determined by the extent to which theories and explanations improve on existing knowledge and have not yet been shown to be obsolete by further scientific progression (Bereiter, 1994). In this sense, science itself is constructed, and in practice aligns more closely with constructivist approaches to

Knowledge Building than to a notion of a learnt body of facts or truths. “Major scientific advances come about when a new theory not only accounts for existing facts but generates predictions that result in new facts that the new theory accounts for but that older theories do not” (Bereiter et al., 1997, p. 331). The purpose of science is to work to replace existing theories with those that are more robust; in other words, those that can account for the same understandings, concepts and phenomena as previous theories, but that can do so more efficiently, more accurately, more elegantly, or perhaps more extensively, and a vehicle for achieving this within the scientific community is discourse.

In a Knowledge Building paradigm, discourse takes on a crucial role. If we accept Popper’s assertion that not only can we not know ‘truth’ we cannot even know if we are getting close to it, then we have no objective viewpoint from which to determine any form of scientific progress. In this sense, discourse can provide that means:

> The importance of discourse to scientific progress was brought out strongly by Karl Popper and further developed by Imre Lakatos. Its importance arises from a recognition that scientific theories cannot be verified; they can at most be falsified. Progress therefore arises from continual criticism and efforts to overcome criticisms by modifying or replacing theories. Research, according to this view, does not generate progress directly, but does so by providing evidence that can be brought into the critical discourse, where it may lead to progress. The prospect offered is one of endless discourse, provided that opposing views are brought into the discourse. (Bereiter, 1994, pp. 5-6)

Essentially, we must ask, within the community: Does a new theory explain facts better than an old theory? Does it explain facts that the old theory could not? Can it explain facts more simply? In this way, scientific knowledge has potential to progress (Bereiter et al., 1997, p. 332). And if there is disagreement about the new theory being an improvement then discussion, argumentation, and further research are indicated. Bereiter’s assertions underscore the essential nature of situating that discourse within a scientific community.

Brown, et al. (1989) depict learning as a process of enculturation: as an apprenticeship into a *community and its culture* (p. 34). The process of enculturation serves to inculcate or induct students into the behaviours, language, ways of knowing, or in short, the epistemology of a discipline, and enables students to act within the normative boundaries that have been accepted by members of the discipline past and present. This is a vital process not promulgated through explicit teaching but rather *is a product of the ambient culture* (J. S. Brown et al., 1989, p. 34). However, in the classroom this culture is rarely apparent; instead, the culture is that of school mathematics in which students use and apply formulae that are very different from the ways they are applied by practitioners of mathematics (Schoenfeld, 1991).

Advances in knowledge require discourse within a community of practice that is grounded in communicative and social activity (Lave & Wenger, 1991). Bereiter et al. (1997) describe what it means to be a community committed to scientific progress by describing four essential ‘commitments’ the community, and its members, must make:

1. *Mutual advances in understanding*: Similar to the principles of epistemic argumentation (Biro & Siegel, 1992; Lumer, 2010; Siegel & Biro, 1997), the purpose of Knowledge Building discourse is to create understandings or theories that are acceptable to all involved. It is not to establish agreement for the sake of agreement, or through submission, but rather “to achieve something that all persons agree is an improvement over their own previous understanding.” (Bereiter et al., 1997, p. 333)

2. *Empirical testability:* Questions or hypotheses proposed require framing in a manner that makes it possible to address them through the obtaining of evidence. The evidence considered acceptable is also a matter for agreement by the community. Bereiter et al. stress that this also means “voluntarily making your position vulnerable” (1997, p. 333). Essentially it is necessary to present all relevant evidence in order for the community to evaluate that evidence, as distinct from obfuscating, selectivity, use of persuasive devices, and so forth, in order to ‘win’.

3. *Expanding the basis for discussion:* This commitment works to enhance the possibility for constructive argument by identifying accepted or agreed upon aspects in order to highlight opportunities for argument around disputed aspects.

4. *Openness:* Finally, participants need to be open to having ideas challenged and disputed, and to considering possibilities beyond their own and even those of their group to achieve “a new mutual understanding” (Bereiter et al., 1997, p. 333).

Commitment to these four principles dictates a guiding role in the research undertaken and reported here in significant ways. The first is that these four principles guide the entire research focus. The nature of argumentative practice, as implemented here, seeks to develop pedagogical approaches to teaching and learning mathematics that incorporate these commitments. This is characterised by classrooms in which knowledge is seen by students and teachers to be a collaborative and mutual advancement in understanding, established through sharing of ideas and understandings. This sharing must be characterised in turn by acceptance of alternate ideas and openness to providing challenge, being challenged, and identifying those areas that need to be addressed further in order to develop deeper and more accurate or acceptable understandings. The classroom must also be characterised as one which values evidence and recognises the role of the community in determining the acceptability and quality of evidence.

Commitment to these four principles also drives the research design. In this instance the aim of this research was not to put forth iron-clad theories or to assert ‘best practice’. Rather, it was to develop pedagogical theory of inquiry-based argumentation in mathematics: a humble attempt to begin shedding light on an area the researcher feels may have potential for use as a pedagogical approach. As such, the work undertaken here is open to challenge and dispute and actively seeks to provide a basis for discussion within the educational research community, with an end goal of advancing understanding within that community.

2.4 Nature of Learning Environments

Adopting a Knowledge Building perspective requires a shift towards a learning community as distinct from one that is either student or adult dominated, or one that is merely a collection of individuals working towards an individual goal. The work of Brown and Campione (A. L. Brown & Campione, 1996) into Fostering Communities of Learners (FCL) has been explored and, with limitations, demonstrates theoretical compatibility with Knowledge Building (Scardamalia & Bereiter, 2013). Collaboration between Knowledge Building approaches and Brown and Campione’s Fostering Communities of Learners model (FCL) has presented some difficulties in practice. However, these are largely centred on smaller differences rather than deeper underlying principles (Scardamalia & Bereiter, 2013). The intent in the research described here was to draw on the compatibility of principles to guide the research design and practice.

2.4.1 Fostering communities of learners

Rogoff, Matusov, and White (1996) describe three models of teaching and learning; adult-run, child-run and community of learners. The notion of adult-run education corresponds to a theoretical approach consistent with knowledge transmission models. These focus on the passing down of knowledge from the expert (the teacher) to the novice (the child). Children-run models align with placing the responsibility for learning with the child who acquires knowledge through exploration or discovery activities as established by the teacher. It would be simplistic to consider these as two extremes as there are varying degrees of control and responsibility for learning afforded under each model; however, in each case, learning is viewed as inherently one-sided. Conversely, a community of learners involves active learners and skilled partners who provide leadership and guidance: "correspond[ing] with the theoretical stance that learning involves transformation of participation in collaborative endeavour" (Rogoff et al., 1996, p.388). A contrast of significant aspects of these models is provided in Table 2.2.

A community of learners' model should not be considered 'middle ground' between adult-run and student-run models. Nor should it be considered a 'flexible' version that justifies an eclectic mix of activities and methods. At times there is flexibility in that adults may at times provide strong leadership and direct guidance and teaching, while at other times the students make progress decisions (Rogoff et al., 1996). However, these practices take place in the nature of "making choices and solving problems in ways that fit their individual needs while coordinating with the needs of others and with group functioning" (Rogoff et al., 1996, p. 405). Thus, decisions made about classroom practice and procedures, such as participant roles, logically seek to maximise the community function.

Table 2.2: Comparison of adult-run, student-run and community models of learning (Rogoff et al., 1996, pp. 393-395)

	Adult-run	Student-run	Community of Learners
Teacher's role	Prepare the knowledge for transmission Subdivide tasks into small mechanical units Determine optimal timing of instruction	Set up the learning environment Avoid being an impediment to learning Passive source of materials	Contributing to the direction of the activity Providing children with guidance and orientation Attentive to what children are ready for and interested in
Student's role	Be receptive, cooperative	Actively construct knowledge Be an active agent in learning	Actively manage their own learning Co-ordinating with adults Providing adults with guidance and orientation
Theoretical underpinning	Knowledge and skills progress from those who have them to those who don't	Children discover knowledge on their own or through interaction with peers	Children and adults collaborate in learning endeavours
Motivation	Applying incentives (or threatening punishment) for students to get through work Teacher motivates students to make themselves receptive	Natural course of learning	Dynamic and complementary group relations among members who take responsibility for their contribution to their own learning and to the group's functioning
Discourse	One-way with children's contributions considered as interruptions	One-way with teacher considered a passive resource	Conversational: people build on one another's ideas on a common topic guided by the teacher's leadership
Assessment	Standardised devices to comparatively determine knowledge quantity	Correction may stifle creativity	Emphasis on children's own improvement rather than on comparison with others. Ongoing reflection
Curriculum source	Teacher may not know lesson purpose either as they may be following a curriculum transmitted from their 'experts' higher up	Challenge is to get the 'natural' course of learning to correspond with the skills and standards valued by the community	Topics of interest to the community

2.4.2 Knowledge Building in learning

Perhaps the most efficient way to address Knowledge Building is to identify the underlying principles that characterize Knowledge Building practices. Scardamalia (Scardamalia, 2002; Scardamalia & Bereiter, 2010) identifies twelve such principles as follows:

1. *Real ideas and authentic problems.* Ideas are considered real entities - as real as concrete objects – students are concerned with understanding, based on their real problems in the real world.
2. *Improvable ideas.* All ideas are treated as improvable – giving students' license to advance unformed or un-evidenced ideas which they can then seek to improve.
3. *Idea diversity.* Diversity of ideas raised by students is necessary in order to contrast, compare, reconceptualise, recreate and extend concepts.
4. *Rise above.* Students learn to work with messiness, diversity and complexities and work with or transcend these to create something organised or improved.
5. *Epistemic agency.* Students themselves take on many of the roles that might normally be undertaken by a teacher: planning, goal-setting, negotiating between their own ideas and understandings and those of others.
6. *Community knowledge, collective responsibility.* Individual focus is diminished in favour of joint responsibility for collective knowledge advancement of the community.
7. *Democratizing knowledge.* All individuals legitimately contribute to the advancement of knowledge in the classroom.
8. *Symmetric knowledge advancement.* All groups give and receive knowledge – the aim is for everyone to advance rather than for one more knowledgeable group to pass knowledge to a less-knowledgeable group.
9. *Pervasive Knowledge building.* All tasks and activities, in and out of school, represent opportunities for knowledge building.
10. *Constructive uses of authoritative sources.* Engagement with the discipline means familiarity with the current state of knowledge, the acceptable sources of this knowledge, and appropriate respect, understanding and critical approach towards them.

11. *Knowledge building discourse.* “Knowledge itself is refined and transformed through the discursive practices of the community – practices that have the advancement of knowledge as their explicit goal” (Scardamalia & Bereiter, 2010, p. 10)
12. *Concurrent, embedded, and transformative assessment.* Assessment is used to advance knowledge. It is used to make judgements about progression, fine tune knowledge responses, guide decisions and directions.

Essentially, Scardamalia and Bereiter describe a process of knowledge development or knowledge creation that is distinct from a more internalised form of understanding. One of the goals of argumentation is to move students from internalised (and hence unarticulated and unchallenged understanding) towards explanation and ultimately persuasion (Berland & Reiser, 2009). By putting forth ideas as improvable entities, students are able to challenge and build on those ideas as a community, thus treating knowledge as improvable through community argumentation.

Bereiter and Scardamalia (1996) suggest that Knowledge Building in mathematics classrooms “would mean producing the kinds of things mathematicians produce – theorems, structures, algorithms, proofs, along with such subsidiary objects as explanations and justifications” (p. 507). However, there is not an expectation here that teachers will engage students in the same practices as skilled professionals, but rather in developmentally and contextually appropriate versions, without losing the authenticity of the practice (Sawyer, 2006) or the discursive practices potentially embedded therein. It is not necessary that students discover new mathematical theorems to engage in mathematical discourse and practice.

> The fact that classroom discourse is unlikely to come up with ideas that advance the larger discourse in no way disqualifies it. Much of scientific discourse does not break new ground but consists of clarifications, resolutions of doubts, and the like. These will play a large part in classroom discourse. (Bereiter, 1994, p. 9)

What is important is that the new understandings, ideas and concepts are new to the students and that the students can recognise them as an advancement over the previous understandings (Bereiter, 1994).

There is yet another level of ‘mathematician’ that is not specifically addressed in the quote above (but may have been intended). Bereiter and Scardamalia (1996), in their reference to proofs and theorems, may be considering professional (pure) mathematicians only, or also those who employ mathematics in other fields of endeavour, including those who use mathematics during the course of day-by-day to day living. Thus ‘mathematicians’ could be argued to include engineers, logisticians, loan officers, and nurses, along with those who engage in grocery shopping, banking, driving, cooking: essentially everyone. This has implications for teachers adopting and adapting authentic contexts and activities to the learning environment. In order to learn these subjects (and not just to learn about them) students need much more than abstract concepts and self-contained examples; they need to be exposed to the use of conceptual tools in authentic activity and to teachers acting as practitioners and using these tools to wrestle with problems of the world (J. S. Brown et al., 1989, p. 34).

Lerman (2001) would not necessarily agree, arguing that the school-based knowledge building community is separate from all of the social practices, goals, and purposes, which are a part of work practices. He argues that school woodwork cannot be carpentry, that school physics is not physics and so forth. However, this stance warrants challenging to some extent: rather than seeing carpentry and school woodwork as disparate, or for our purposes, the practice of mathematics and school mathematics, these practices could be interpreted as existing along a continuum. While it may not be possible to replicate the authentic practice exactly, there are multiple benefits for approximating activities that are close to authentic practice. Furthermore, many of the social contexts of the classroom mimic the social constraints of the workplace: dominant culture, gender bias, social relationships. While Lerman also makes reference to the constraints of assessment practices, perhaps the challenge is to relate those assessment practices as much as possible to the authentic task and the authentic workplace, with its inherent social practices.

2.4.3 Classroom culture

If the underlying principles of both Fostering Communities of Learners and Knowledge Building practices are addressed, it becomes evident that the two have commonalities which would enable co-existence in a classroom (Scardamalia & Bereiter, 2013). For instance, in a Community of Learners, students would be anticipated to co-ordinate with adults to manage their own learning: a concept quite compatible with the Knowledge Building principle of joint responsibility for collective knowledge advancement. In addition, given the essential nature of discourse in argumentation practices, again we see compatibility between Knowledge Building, in which those discursive practices aim to advance knowledge, and in Communities of Learners, where a guiding principle is to utilise discourse to enable community members to build on the understandings of other members. With respect to this last point, however, Bereiter and Scardamalia argue there is a lack of compatibility with the teaching of mathematics itself, as mathematics in schools, or indeed the wider community, is rarely seen to be an improvable construct:

> School mathematics has scarcely anything to do with …the world of mathematical ideas treated as improvable human constructions. Instead it is occupied with the individual acquisition of skills, supplemented to a greater or lesser extent by activities involving …objects, known as “manipulatives”. (Bereiter & Scardamalia, 1996, p. 505)

In authentic practices of mathematics, the practitioner will often be a part of a larger working community, particularly in workplace settings.

> Studies of knowledge workers show that they almost always apply their expertise in complex social settings. …These observations have led learning sciences researchers to a situativity view of knowledge. “Situativity” means that knowledge is not just a static mental structure inside the learner’s head; instead, knowing is a process that involves the person, the tools and other people in the environment, and the activities in which the knowledge is being applied. (Sawyer, 2006, p. 4)

Therefore, it is important that students have opportunities to engage in such authentic community practices to provide experiences in articulating their understandings, thought, ideas, conceptions and so on. Building a community of learners in mathematics would require a significant pedagogical shift for the majority of teachers. However, it also requires a significant cultural shift for both students and teachers if the classroom had been based on traditional approaches. For example, teachers may need to take on more expansive and varied roles in this context, such as “mediator, diagnostician, modeller, and collaborator” (Crawford, Krajcik, & Marx, 1999, p. 720). Students also develop a reliance on teachers as the arbiters and bearers of knowledge and developing a deeper reliance on themselves and their own ability to negotiate problems requires a level of risk-taking and resilience. A classroom culture in which students respect and build on each other's ideas is required.

This leads to the final consideration to be addressed here, and that is the nature and role that context plays in learning. It is difficult to conceive of a classroom in which students engage with, challenge, and build on the ideas of others, if the mathematical context is uninteresting, unchallenging or perceived as meaningless or contrived.

2.5 Nature of Role and Context in Learning

Authentic activities can be considered as those activities that ordinarily take place within the practice of a discipline or domain: "the ordinary practices of the culture" (J. S. Brown et al., 1989, p. 34). In the case of mathematics, the domain of the mathematics may vary greatly, as mathematics can be practiced through both day-to-day living and in a profession; for example, by engineers, project managers, biologists, cartographers, geologists, aircraft refuelers, nurses, builders and so on. In each of these occupations, mathematics is used in very different ways, from formal and field independent, through to highly informal and deeply contextualised.

One advantage of addressing situated problems is that the context itself, and the tools appropriate to the context, can serve as a support. One example provided by Brown et al. (1989) was that of a dieter wanting to find three-quarters of two-thirds of a cup. Rather than calculating 3/4 x 2/3 as an algorithm, he measured 2/3 of a cup, divided it into four parts and removed one quarter. The cup measure provided the tool to approach the problem concretely, quickly and efficiently. This contextualised, situated thinking also enables students to develop informal understandings prior to engaging with a subject formally, and enables more complex problems to be addressed meaningfully by younger students.

However, the nature of mathematics teaching in schools is such that mathematics is often decontextualized, with mathematical concepts made highly abstract: "the activity and context in which learning takes place are …regarded as merely ancillary to learning" (J. S. Brown et al., 1989, p. 32) and this, in turn, has the potential to strip away the contextual features necessary for authentic engagement. It is the removal of these peripheral features of authentic tasks when creating classroom tasks which also removes authentic supports.

Ironically, it is not uncommon to see contexts strategically applied to create classroom (textbook) problems. This is ostensibly done to prevent distraction, while simultaneously offering a potential problem context, to do so is an injustice to the discipline, leaving students with the impression that mathematics is artificial and contrived. Brown et al. (1989) argue that

> by participating in such ersatz activities, students are likely to misconceive entirely what practitioners actually do. As a result, students can easily be introduced to a formalistic, intimidating view of math that encourages a culture of math phobia rather than one of authentic math activity. (p. 34).

Schoenfeld (1991) suggests that students will actively seek to work within a context, even if one is not overtly offered. If the context is school mathematics, the students will still seek to identify and use supports: the textbook structure, the knowledge of teaching practice and examination methods and so forth. Students know that the chapter heading in the text book will give them the method, that the easiest questions are asked first, and that a worked sample will be in the book somewhere within a few preceding pages of the question they are addressing. In this way, students “may come to rely, in important but little noticed ways, on features of the classroom context, in which the task is now embedded, that are wholly absent from and alien to authentic activity.” (J. S. Brown et al., 1989, p. 34). This reliance on this ‘school’ cueing can result in fragile learning (Schoenfeld, 1991).

Clearly there are restrictions to the degree of authenticity that may be transferred to a classroom; for example, students cannot build a house in order to authentically undertake the mathematical work of an architect or builder. However, they could design, plan, cost, and construct a doll’s house for the Kindergarten class. Such activities simulate similar principles while enabling students to make connections to the real-world activities mathematics practitioners undertake. The inclusion of an authentic context offers potential to motivate students towards Knowledge Building practices: an important consideration given that motivating towards Knowledge Building in school mathematics appears to be more difficult than in empirical domains. For instance, Bereiter and Scardamalia found that students could

> generate productive and challenging questions when invited to state what they wonder[ed] about in all sorts of areas of natural and social science. But these same students tend[ed] to be dumbfounded when asked what they wonder[ed] about in mathematics. They [did] not see that there [was] anything to question or wonder about with regard to mathematics. (Bereiter & Scardamalia, 1996, p. 507)

Two possibilities are suggested as a means to overcome this problem (Bereiter & Scardamalia, 1996). The first is to introduce more real-world situations, or 'stories' constructed around the real-world, into the classroom. "Although this approach may motivate more extensive and meaningful uses of mathematics as a tool, it is not clear that it brings students any closer to the construction of [their own theorems and conjectures]" (p. 507). A second approach is to make numbers themselves interesting in their own right, rather than as representations or attributes of physical things.

Ainley, Pratt and Hansen's (2006) work into pedagogic task design addresses some of the difficulties that may arise from attempting to introduce real-life or authentic tasks into the classroom; specifically, the potential for a less-focussed approach to mathematics and a related difficulty in monitoring actual learning. In response, they propose a framework which encourages teachers to address *purpose;* that is, that a task should have *"*a meaningful outcome for the pupil", and *utility,* essentially that it leads to the development of meaning for the ways in which mathematical ideas are useful (pp. 29-30). These principles have been applied to the design of the tasks as implemented in the research addressed here, and this is discussed in the Methodology.

2.6 **Conclusion**

In this chapter, a brief overview has been provided of the underlying philosophical positioning of the author, and the author's research. The principles surrounding the nature of scientific knowledge as constructed through discourse and verisimilitude have been addressed, along with the implications this has for learning environments. In order to seek to develop a learning environment that values such an approach, the principles of Knowledge Building, and Fostering Communities of Learners that were drawn upon have been described. Finally, attention was given to the classroom culture and task contexts that would be considerations of importance when implementing such a philosophy of teaching and learning. This is perhaps best summarised in Scardamalia and Bereiter's own words (2006):

> Creative knowledge work may be defined as work that advances the state of knowledge within some community of practice, however broadly or narrowly that community may be defined. Knowledge building pedagogy is based on the premise that authentic creative knowledge work can take place in school classrooms – knowledge work that does not merely emulate the work of mature scholars or designers but that substantively advances the state of knowledge in the classroom community and situates it within the larger societal knowledge building effort. This is a radically different vision from contemporary educational practice, which is so intensely focused on the individual student that the notion of a state of knowledge that is not a mental state seems to make no sense. Yet in knowledge creating organisations it makes obvious sense. People are not honoured for what it is in their minds but for the contributions they make to the organisation's or the community's knowledge. (pp. 98-99)

The chapter following will provide background literature on one pedagogical approach – argumentation – that potentially could be implemented in a manner compatible to the philosophical underpinnings of Knowledge Building and Communities of Learning. As argumentation has been the focus of significant research in science education, much of the research comes from that area.

3 Literature

3.1 Chapter Overview

This literature review serves to set the scene for the research undertaken and reported here by providing an overview of argumentation and argumentation practices. Various definitions of argumentation exist in the literature and it is necessary to explore these definitions around the themes of structure, goal and purpose. Consideration is given to previously researched benefits of introducing argumentation as a pedagogy in science education in order to justify research into its use in mathematics education. While the research reported here does not extend to explicitly identify learning outcomes of argumentation pedagogy in mathematics, the potential of some of these become incidentally identifiable in the results sections.

Traditional ways of teaching do not provide a classroom culture that is necessarily conducive to the introduction of argumentation practices and thus there are many practical considerations to developing such approaches. In order to facilitate the research undertaken, argumentation was introduced into primary classrooms that were already fluent in the use of inquiry-based learning (IBL) in mathematics. Accordingly, consideration is given to the nature of IBL in mathematics and the reasoning for using IBL as a setting in which to implement argumentation. Associated issues such as the consideration of support for teachers and the development of an argumentation culture are also addressed in terms of existing literature. Finally, the essential elements of planning IBL tasks for this purpose is considered along with student specific factors, such as developmental readiness for engaging in argument.

3.2 Introduction

Argumentation as a construct is complex and multi-facetted, with significant ongoing critical debate taking place across many aspects of argumentation theory. While the research described in this document does not intend to add to that debate, nor advance propositions in terms of those arguments, it is essential that the broader of these be considered in order to provide clarity to this research, and to situate the reader accurately. The debate particularly centres around several broad points of contention: the nature of the relationship between persuasion and argumentation; agreement of what constitutes support for an argument; and, the generic nature, purpose, and structure of argument. Essentially, there '"is no correct (or universally-endorsed) definition of either 'persuasion' or 'argumentation'. … Definitions of argumentation systematically vary in two ways, namely, (a) the communicative ends specified and (b) the communicative means specified." (O'Keefe, 2012, p. 20).

The first of these points centres on the place of persuasion in argumentation theory and the extent to which persuasion and argumentation overlap. Is persuasion the goal of argumentation? Are they similar and compatible? Should they be carefully delineated (Nettel & Roque, 2012). However, argumentation in the contemporary scientific sense is quite different from a lay definition, which is largely to persuade or 'win'. Rather than aiming to achieve a 'winning' position, argumentation in the scientific sense involves collaborative discussion to explore and resolve an issue in order to construct an explanation which best fits available evidence and logic (Berland & Reiser, 2009). The focus of such an endeavour is on "collaboration not competition" (Sampson & Clark, 2008, p. 296) with argumentation considered "a social and collaborative process necessary to solve problems and advance knowledge" (Duschl & Osborne, 2002, p. 41).

According to Nettel and Roque (2012) there is a second debate, related to the first, which is centred

> on the means used within argumentation and persuasion. It concerns roughly the model of rationality put forward by the theories of argumentation. As the evaluation of arguments rests on rationality, or reasonableness, the question to be addressed is whether or not the rational means of argumentation are compatible with persuasion? Is persuasion rational, and if so how? Or do we have to exclude from argumentation all which is considered as ''influence'' or ''suggestion''. This dispute is also often focused on the opposition between reason and emotion, the former being associated with argumentation and the latter with persuasion. (p.2)

These views are largely influenced by the context in which the argumentation practice is taking place. Argumentation in the advertising context often leans far more heavily to the persuasive side than reasoning, although a factual basis may be present. Scientific research by contrast aims to provide a dispassionate approach, characterised by the collection and analysis of valid and reliable data, and the careful and cautious interpretation of such. Within scientific research, it is the methodology adopted and the analysis and interpretation that may be the site of the argumentation. However, science often engages in such research in order to address wider practical issues, and this purpose presents a further site for argumentation: that of the application of scientific endeavour to actual issues, and this may lead to the incorporation of more emotive and persuasive components.

3.3 What is Argument? Argumentation?

In order to undertake any study of argumentation, it is important to first explicate what is meant by the term. Multiple theories of argument have been put forth, and some of the more widely utilised are discussed below to frame the decision to adopt epistemic argumentation in practice (Biro & Siegel, 1992; Lumer, 2010; Siegel & Biro, 1997).

Argument essentially exists in two forms; it is both a product and a discursive practice (Leitão, 2000; O'Keefe, 1977). While some use the term ‘argumentation’ to collectively refer to both (Rowland, 1987), Blair defines argument as “a set of one or more reasons for doing something” and argumentation as “the activity of making or giving arguments” (2012, p. 72). In all cases, the argument essentially includes a statement of position (claim or thesis) and supporting grounds for that position. In terms of process, the argument itself occurs through the act of delivery.

The most common view of argument, and the widely accepted lay position, is a model of confrontation, whereby each side makes claims, defends them and argues against any opposing claims until a ‘winning’ position has been established. By default then, there must also be a losing position. A characteristic example being that of the classic debate, where both sides are able to take turns in championing their theses, arguing the opposing position and defending their own until a winner is declared. The majority of ‘lay’ people would be able to describe an argument if asked, and would usually describe it by this common purpose, the achievement of a ‘win’, or the persuading or convincing of others to carry out an action. These actions could include performing a particular action; or it could be encouraging thinking in specific ways, or a taking on of particular beliefs, which may or may not have a resultant action.

However, this confrontational view of argumentation is largely inadequate and one which is limiting. In reality, the dimensions of argument and argumentation are more complex. For example, arguments may be spoken or written; they may involve multiple parties such as in a political debate or judicial hearing, or may be rhetorical as is often the case with advertising or a sermon. The persuasive devices used may range from scientific 'truths' through to emotive opinion: while the intent to convince ranges from clearly overt to that which is carefully and purposefully obscured. The argument may take place on an intrapersonal level, interpersonal level, or be conducted with a formal or an informal audience. The intent of the argument process may also vary greatly, ranging from a desire to seek objective truth through to eristic argument or sophistry. Argument also serves to enable both simple and complex decision-making and problem solution. Thus, even the purpose of argument can be seen to be a source of disagreement among theorists.

In the 1990's, van Eemeren and Grootendorst attempted to refine some of these issues through their seminal *pragma-dialectical* approach to argumentation, in which the practical purpose of an argument is held to be to reach consensus; that argumentation should be seen as a "rational means to convince a critical opponent and not as mere persuasion. The dispute should not just be terminated, no matter how, but resolved by methodically overcoming the doubts of a rational judge in a well-regulated critical discourse" (1992, pp. 10-11). van Eemeren and Grootendorst (2004) detail what they mean by critical discourse:

> A critical discussion can be described as an exchange of views in which the parties involved in a difference of opinion systematically try to determine whether the standpoint or standpoints at issue are defensible in the light of critical doubt or objections. Unlike, for instance, formal dialectics, our approach to argumentation is not only dialectical, but also pragmatic. The pragmatic dimension of our approach manifests itself primarily in the fact that the moves that can be made in a discussion aimed at *resolving a difference of opinion* [emphasis added] are conceived as verbal activities … carried out within the framework of a specific form of oral or written language use. (p. 52)

Under the pragma-dialectical model, the difference of opinion is considered resolved under two conditions: if all parties come to an agreement about the opinion or if the protagonist withdraws his viewpoint (van Eemeren & Grootendorst, 2004, p. 133). This essential nature of consensus under the pragma-dialectical model gave rise to challenge in that critical reasoning should be deemed more important than consensus as consensus does not guarantee the most accurate, most correct, or the best possible solution (Lumer, 2010). There are a multitude of other factors that could lead to consensus that would lack rigor, including; emotive positioning, trickery, the likelihood of one party conceding to ‘keep the peace’, eristic manoeuvres (Lumer, 2010), or false analogies, circular arguments or any of the other fallacious grounds identified by Toulmin (1958).

To address this, Seigel and Biro (1997) and Lumer (2010) proposed *epistemic argumentation*. Epistemic argumentation theories propose argumentative discourses as those which collectively seek the truth through critical reasoning and justification (Lumer, 2010). While the goal is to reach consensus:

> it is a qualified, justified consensus, where both parties not only share the final opinion but—ideally—also their subjective justification for it. To take justified consensus as the aim of argumentative discourse avoids all the problems listed so far because justification—correctly conceived—is related to truth. It is based on cognizing procedures that guarantee the truth or at least the acceptability, i.e. truth, high probability or verisimilitude, of the results. (Lumer, 2010, p. 48)

While both pragma-dialectical and epistemological models agree that the focus of argumentation is on “collaboration not competition” (Sampson & Clark, 2008, p. 296), epistemological argumentation distinguishes itself by evaluating the strength and validity of an argument through epistemic criteria only (Nettel & Roque, 2012). Epistemic argumentation therefore more closely aligns with the philosophical underpinnings of advancing knowledge-building practices.

For the purposes of this thesis then, epistemic argumentation is adopted and the term ‘argumentation’ will be “used to refer to the whole activity of making claims, challenging them, backing them up by producing reasons, criticising those reasons, rebutting those criticisms, and so on” (Toulmin et al., 1984, p. 14); while ‘argument’ will refer to the product, that is, the claim and supporting grounds. The term ‘reasoning’ is intended to describe the inclusion of reasons in support of a claim, so as to show how provided grounds support a claim.

3.4 **Rationale for the Use of Argumentation**

There are multiple potential benefits to introducing argumentation within the classroom. Much of this research is well-documented, though situated predominantly in science education, where both IBL and argumentation have been a focus of research for some time. If a premise is accepted that there are parallels between the nature of science and scientific learning, and mathematics and mathematical learning, then there exists a likelihood that benefits apparent in one domain may be of potential benefit to the other; however, this remains to be tested.

Jimenez-Aleixandre and Erduran (2007, p. 5) propose five benefits from the introduction of argumentation into the science classroom, drawn from various bodies of knowledge, and these are represented in Table 3.1. These, they argue, are not separate but rather intertwined; however, for the sake of discussion they will be addressed separately.

Table 3.1: Potential contributions of argumentation to the science classroom

Potential Contributions of Argumentation	Drawn From (as identified in Jimenez-Aleixandre & Erduran, 2007)
Supporting the access to the cognitive and metacognitive processes characterising expert performance and enabling modelling for students.	The situated cognition perspective and the consideration of classrooms as communities of learners (A. L. Brown & Campione, 1990; Collins, Brown, & Newman, 1989)
Supporting the development of communicative competencies, particularly critical thinking.	The theory of communicative action and the sociocultural perspective (Wertsch, 1991)
Supporting the achievement of scientific literacy and empowering of students to talk and to write the languages of science.	Language studies and social semiotics (Kress et al., 2001; Norris & Phillips, 2003; Yore et al., 2003).
Supporting the enculturation into the practices of the scientific culture and the development of epistemic criteria for knowledge evaluation.	Science studies, particularly from the epistemology of science (Leach et al., 2003; Sandoval, 2005).
Supporting the development of reasoning, particularly the choice of theories or positions based on rational criteria.	Philosophy of science (Giere, 1988; Siegel, 1989, 1995, 2006) as well as from developmental psychology (Kuhn, 1991, 1993).

The first of the benefits of argumentation practices in the classroom is that of enabling access to the cognitive process of each other. A significant difficulty with transmission methodology is that neither students nor teachers have access to the other's thinking and reasoning. The framework of argumentation practice is such that it provides opportunities to call a viewpoint into question. The exchange of opposing views, grounds and supporting reasoning gives the audience and the proponent the opportunity to examine their own conjectures, thoughts, and understandings, and thus emphasises cognitive and metacognitive processes (Leitão, 2000). As cognitive processes are made public through argumentative discourse (including written inscriptions and artefacts, such as diagrams and models tendered as evidence), teachers are able to appreciate and enhance accurate student understandings and identify and deal with immature conceptions (Jimenez-Aleixandre & Erduran, 2007). Furthermore, argumentative discourse enables the 'visibilising' of the teacher's thinking. Thus the teacher is able to model the thought processes of the practitioner; enabling the inculcation of students into the cognitive and discursive practices of the discipline. Such discursive interaction contributes to a second benefit: the development of communicative competencies.

Argumentation practices necessitate the assumption of an Other; the audience with whom one engages in dialogue, or to whom the dialogue is addressed. "The conversational presence of the other is a critical factor that must be taken into account for the development of argumentative skills" (Pontecorvo & Pirchio, 2000, p. 363). Argument does not of itself require an actual Other, one may actually address oneself and tacitly construct reasons, oppositions and counter-arguments in dialogue with oneself as a 'virtual' other. However, the virtual other must necessarily be acknowledged even under those circumstances (Leitão, 2000; Pontecorvo & Pirchio, 2000) as the role assumed is not essentially monologic; opposing views do not necessitate opposing individuals. The necessity to convey a message clearly and to articulate reasoning requires complex communicative skills. Advanced expressive and receptive language skills are necessary, particularly when consensus is not immediate. The arguer must consider how best to convey a message to achieve clarity and minimal loss of meaning, while also paying careful attention to the reception of others' responses. This is most demanding in dialogic argument (due to the presence of a responsive audience).

At the language level, students are provided the opportunity to develop high levels of subject specific literacy, to be able to talk and write within the language of the subject (Jiménez-Aleixandre & Erduran, 2007): essential skills as future operators, constructors and consumers in a globalised society (Skovsmose, 2008). Beyond the language level, the use of argumentation promotes enculturation into the practices and discourse that are inherent in the study and application of science and mathematics. Explanation and argumentation within and across subject domains assist with the enculturation of students into a subject. Articulating to a critical audience necessitates considerable strength of the argument. In scientific argument, this strength is centred in disciplinary knowledge – that which is acceptable as knowledge within the discipline. Furthermore, the ability to challenge the argument is offered on an epistemic level, giving potential rise to challenge about what is acceptable evidence and reasoning within a discipline (S. Simon & Richardson, 2009).

> A view has now become established that argumentation is a central practice in science and should thus be at the core of science education, and that understanding the norms of scientific argumentation can lead students to understand the epistemological bases of scientific practice. (S. Simon & Richardson, 2009, p. 470)

Research suggests that increasingly secure conceptual understandings can occur when students have opportunities to work with both accounts of a phenomena and its associated evidence (Howe & Mercer, 2007), particularly through the development of reasoned arguments either in favour of, or contrary to, one’s own views (Asterhan & Schwarz, 2007). Consensus is largely established that argumentation enables the building of new knowledge and the changing of beliefs and/or views (Forman, et al., 1998; Leitão, 2000; Orsolini & Pontecorvo, 1992).

The final benefit of argumentation identified by Jimenez-Aleixandre and Erduran (2007) was to support the development of reasoning, particularly the choice of theories or positions based on rational criteria. Through the use of argumentative discourse, students have greatly increased opportunities, and can be supported, to develop explanation, debate, justification and defence while engaged in the practices of reasoning, explanation and persuasion. However, “because argumentation always aims at getting an addressee to accept reasons and positions that relate to controversial matters, it also requires the arguers to examine their claims in the light of opposing claims of others" (Leitão, 2000, p. 335). Hence why many corporate groups appoint 'devil's advocates' when making significant decisions – the presence of competing or dissenting values can deepen the level of reasoning significantly. The very nature of argumentative discourse is such that it requires reasons to be put forth in order to establish and support an epistemically justifiable position; thus, it can be argued that argumentation practices encourage students to select and defend their choice of theories or positions based on rational criteria.

Given this body of literature details researched benefits to scientific argumentation as a teaching pedagogy, it is appropriate to question why widespread usage is not evident in the classroom and more strongly through the mathematics curriculum. One reason may be the perception that mathematics is a fixed body of fact. In terms of pure mathematics, while mathematical proof can be regarded as a species of argument (Aberdein, 2009, p. 1), there is a great deal that mathematicians also do that incorporates reasoning and argument within the frameworks described by Toulmin et al. (1984). For example, factors leading to choice of problems to study, selections of methods used, and the means of applying a method to a problem are all open to challenge and defence (Aberdein, 2009). Further, even pure mathematical argumentation, in terms of proof, must still stand up to rigorous, critical, dialectical argument by other mathematicians and be open to argument as attempts are made to examine, generalise, extend, and simplify the proof. Significantly though, the majority of users of mathematics are not pure mathematicians, but rather those who apply mathematics in their daily lives and work. Mathematics is therefore heavily contextualised and this use of context increases the opportunities and applications for argumentation.

Lave's study of everyday activity (1988) addressed the learning of everyday people, or 'just plain folk' (JPF) and detailed differences between 'apprenticed' learning and school learning. Apprenticed learning was characterised by issue resolution, meaning negotiation and intuitive reasoning, while school learning adopted well-defined problems with procedural guidance and formal definition. Brown et al. represented these differences reproduced here in Table 3.2 (J. S. Brown et al., 1989, p. 35). The purpose of the table is to draw attention to the similarities between the activities of JPF and Practitioners as distinct from the activities of students, highlighting the disparate nature of students' activities from those conducted in the ‘real’ world and perhaps hinting at the inappropriateness of such learning in preparing students for life outside school.

> Instead of taking problems out of the context of their creation and providing them with an extraneous framework, JPFs seem particularly adept at solving them within the framework of the context that produced them. This allows JPFs to share the burdens of both defining and solving the problem with the task environment as they respond in ‘real time’. The adequacy of the solution they reach becomes apparent in relation to the role it must play in allowing the activity to continue. The problem, the solution, and the cognition involved in getting between the two cannot be isolated from the context in which they are embedded. (J. S. Brown et al., 1989, p. 36)

Table 3.2: 'Just plain folk', Practitioner and Student Activity (J. S. Brown et al., 1989, p. 35)

	JPF’s	Students	Practitioners
reasoning with:	causal stories	laws	causal models
acting on:	situations	symbols	conceptual situations
resolving	emergent problems and dilemmas	well-defined problems	ill-defined problems
producing	negotiable meaning and socially constructed understanding	fixed meaning and immutable concepts	negotiable meaning and socially constructed understanding

Schoenfeld (1991) describes students strategising in the same way as JPFs, with a reliance on the framework of the context. However, the strategising described by Schoenfeld is specific to ‘school’ mathematics; for example, students relying on the location of problems within the textbook to know which methods or formulae to adopt.

3.5 Developing Classroom Argumentation

Driver et al. (2000) identify two primary impediments to overcome in developing student argumentative practices: the lack of opportunity for argument to be advanced in the classroom, and the lack of teacher skilling in organising argumentative discourse. To this a third can be envisaged, the need for a culture which is conducive to argumentative practices.

3.5.1 Advancing argumentation through Inquiry-Based Learning

Argumentation does not occur in implementing the curriculum for several reasons. Aside from the lack of presence in curriculum documentation, curriculum support materials are not created to foster such an approach. For example, while we have seen some progression in Australia, with the inclusion of *Scientific Inquiry Skills* now a specific strand of the Australian Curriculum (and incorporating explicit mention of students' construction of evidence-based arguments), this is only incorporated at the secondary level, and only with regard to science (ACARA, 2014b). However, scientific text books are often presented in a factual format as distinct from narrative; knowledge is not presented as socially constructed but as absolute, and alternate theories for subject matter are not considered. So while students are not taught written argumentation explicitly, neither are they exposed to the written form incidentally through their reading. In mathematics, the situation may be even more unfavourable, as textbooks typically present little beyond proofs, worked examples, practice questions, and answers. Where questions are contextualised, the problem is typically well-structured and presented as part of a practiced process, that it leaves little room for alternate pathways and no room for alternate answers (Kabiri & Smith, 2003).

Essentially, to implement argumentation practices in mathematics teaching would require the establishment of an environment which is conducive to the exploration of such alternate pathways and alternate answers. To achieve this, an approach which does not treat knowledge as absolute and acknowledges the basis of social construction is necessary. One teaching approach that offers potential to provide such an approach is that of Inquiry-Based Learning (IBL). To better understand this context and its link to argumentation, research on inquiry practices which underpin argumentation is synthesised in the following section.

Over several decades, the term IBL has been used to describe widely varying approaches to student-centred learning: approaches that effectively describe discovery learning (Kirschner, Sweller, & Clark, 2006) through to teacher-guided and managed discussions focussed on closed-ended but open-method problems (Goos, 2004). Despite the length of time that inquiry has been identifiable in research across multiple curriculum disciplines, its initial support and later mandating through the Australian Science curriculum (ACARA, 2014b), and the development of a following in mathematics education, there is still no precise, agreed upon definition of inquiry among researchers. Some describe inquiry in terms of its features, for example, Chu (2008), in a review of IBL literature, describes seven key components of classroom IBL:

- Students are provided with rich information sources.
- Students are equipped with information literacy skills.
- A climate of inquiry is created.
- Scaffolding support is provided to students in developing driving questions.
- Students go through an information-seeking process.
- Students develop their own research process.
- Students learn to present their findings.
- The teacher's role is to scaffold the students' learning through this process and to encourage autonomy.

Others describe IBL in terms of its underlying philosophical approach, such as Cobb, Wood and Yackel (1993) who describe mathematical inquiry as an apprenticeship where ways of thinking are developed within classrooms that encourage and support reflective discourse. Finally others describe inquiry in terms of its intent or aims. For example, McInerney (2006) argues that IBL enables students to experience learning in the real world, stimulating growth of logical thinking and the development of language and deductive thinking. The exposure of students to different points of view, often equally compelling or valid, can bring about cognitive dissonance, with the result that students actively seek to find understanding, potentially explaining why many teachers who have worked with inquiry report an increase in student engagement (H. Brown, 2004). In this process, students are required to defend, justify, modify, concede or relinquish their position, ideas or understandings, necessitating that they accommodate and assimilate developing understandings. The social interaction that takes place during this process is essential for students' cognitive development and interaction with peers through group work. It is also these aspects that make IBL an ideal environment in which to introduce argumentation practices or that which could be referred to as Argumentation-Based Inquiry. However, it should be made clear that this study is not about IBL per se, but is about situating argumentation within a classroom which practiced IBL to facilitate such practice.

3.5.2 Teacher support and skilling

A second impediment to developing classroom argumentation practices as identified by Driver et al. (2000) was a lack of teacher skilling. Argumentation-based inquiry practices differ significantly from traditional, transmission methodologies of teaching, and the implementation of such would require massive shifts in pedagogy from more didactic approaches. Makar (2010, pp. 4-5) found that when teachers were in the early stages of adopting mathematical inquiry into their classrooms; they were largely concerned with three aspects of the inquiry:

- Difficulty with the uncertainties of inquiry – including envisioning the inquiry processes, students' ability to cope with the challenge, negotiating unexpected outcomes and directions, and their changing mindset about learning processes.
- Managing the logistics of inquiry – including classroom management issues, curriculum time constraints, and the balance of student-teacher control.
- A solid content background – deemed necessary for dealing with the uncertainties of inquiry, for helping students to reason, and for guiding them to deeper understandings.

Such concerns and uncertainties may prevent teachers from engaging in inquiry and related argumentation practices. However, Makar's research suggests that, with support, teachers progress through concerns and uncertainties to a level where they demonstrated commitment to inquiry pedagogy: embracing inquiry; creating a classroom culture of inquiry; and, working to engage other teachers in the teaching of inquiry (Makar & Fielding-Wells, 2011, p. 354).

Makar and Fielding-Wells (2011, pp. 355-356) identify two projects in which teachers have been assisted to develop mathematical-statistical investigations and summarise the key characteristics that were identified as assisting: ensuring strong teacher content knowledge; providing opportunities for teachers to themselves engage in investigations as learners; engaging teachers within their own classrooms by having them implement and reflect on investigations; developing collaborative and accountable relationships between teachers and their colleagues; encouraging teacher reflection; and, long-term support and resourcing.

Research from the field of argumentation in science education supports these key characteristics. For example, Simon and Richardson (2009) created training materials which focused on group work strategies; introduction, sustaining and concluding activities; evaluating arguments; modelling and counter-argument in order to enhance argumentative practices. They found that when teachers were supported with these theoretical perspectives and materials, and encouraged to develop their own practices, there was evidence of increased complexity in their argumentation, and provision of more extended arguments (with backings and rebuttals). In addition, the teachers themselves developed increasingly effective ways to scaffold students through the encouragement of argumentative discursive practices, including listening, counter-arguing and reflecting. Osborne, Erduran, and Simon (2004) also found that teachers could be supported to achieve the complex task of envisaging and designing tasks by having them consider their role, articulate goals, think through processes, and envisage how an activity might develop.

There is also a need to support and develop a classroom culture aligned to encouraging argumentation and knowledge building if students have been accustomed to more passive learning. Students are unlikely to be familiar with argument process and structure; at least, not in terms of epistemic argument rather than more formal persuasive discourses such as debate, which they may have formally addressed, but which carries a purpose of ‘winning’ a position. Makar (2010, p. 5) stresses that support needs to be provided to teachers to “develop a culture of inquiry in their classrooms, including explicitly teaching students skills in collaboration, argumentation, and managing project work” and suggests that teachers need to be involved in the process themselves in order to envisage the practice.

Implementing argumentation practices into the classrooms of teachers already practicing IBL was undertaken to bypass many of these phases and to expedite the development of argument for the purpose of this study.

3.5.3 Developing a culture of argument

There is potentially a third impediment that requires addressing, and that is the significant impact of home and classroom culture. Students are not first introduced to argumentation in school: argumentation is an everyday discursive practice. Argumentation practices are developed as a normal course of our way of life, and from a very early age. Take for instance the toddler demanding a lolly, a biscuit, or attention – "*But I want*…", or the child, "*But why can't I go to Sam's, I cleaned up my room and my homework is done?".* In the normal course of development, the reasoning would be expected to become more sophisticated; however, the seeds are there in infancy, and very likely adopted and adapted by children initially watching adults negotiate their world and then later in social interactions with others (Muller Mirza, Perret-Clermont, Tartas, & Iannaccone, 2009). However, Australian children are not usually encouraged in such endeavours, and while adages such as 'children should be seen and not heard' have lost favour, adults still typically expect children to not argue with them and to largely do as they are requested. By the time students reach formal schooling, they have been enculturated into a belief that 'arguing' with authority figures is disrespectful and insolent. In fact, arguing with peers – one's siblings and playmates – is also considered inappropriate: an issue that further impacts on the classroom management practices and balance of control that Makar (2010) observed teachers to be concerned about. It is unsurprising then that students who have little experience with a culture of argumentation are likely to need significant support and scaffolding in order to engage effectively (Jimenez-Aleixandre & Erduran, 2007; Osborne et al., 2004; Pontecorvo & Pirchio, 2000). However, the provision of scaffolding and support is not beyond the means of teachers if they themselves are suitably supported.

3.6 Task Considerations

The nature of the learning task plays an essential role if argumentation practices are to be a classroom focus. In order to create opportunities for argumentation, researchers put forth that it is necessary to purposefully design tasks which generate cognitive differences, that is, alternate theoretical interpretations such as competing theories or opinions, or with anomalous or conflicting data sets (Asterhan & Schwarz, 2007; S. Simon & Richardson, 2009). Another approach is to put students with opposing explanations in opposition so that they are in a position to persuade each other (for example, Bell & Linn, 2000; Osborne et al., 2004). However, in mathematics teaching and learning it is difficult to conceive of putting students in direct opposition by supplying competing theories or methods, particularly at the primary school level, as mathematics as taught is significantly less contextual than aspects of biology, earth sciences, chemistry, or physics for example. Thus it seems that if school-based mathematics lacks the context required for argumentation, it need be provided: in this instance that is done through the use of inquiry questions.

3.6.1 Inquiry-based learning tasks

There are two generally agreed aspects of inquiry and these are that inquiry is typically considered to be the solving or addressing of problems which are both authentic and ill-defined (Anderson, 2002). The rationale is that most problems in life are ill-structured; that is, their problem definition is ambiguous or has many open constraints (H. A. Simon, 1973). A third thread that permeates each of the definitions and approaches is the underlying requirement for participants to engage in socially constructed discourse. The definition adopted in this paper is that inquiry-based learning is the addressing of *authentic*, *ill-structured problems* within *a specific community of learners*.

<u>Authentic Problems</u>

Often when students are posed a mathematical problem, the teacher already has a known 'answer'. As such, students may not engage in an authentic manner as they have no real need to explain or persuade their audience, the teacher, of its validity (Sandoval & Millwood, 2007). However, in IBL, authenticity is an essential component of the inquiry question. First, addressing authentic problems enables them to make connections to the real world, see the potential for mathematics to solve real problems, and have potential to engage students more deeply in a problem (Fielding-Wells & Makar, 2008b). Authenticity may be brought about through the context (providing a problem the students have a real need or desire to answer) or through the audience (by establishing a genuine purpose for reaching a best case scenario). The necessity for authentic problems to be situated in real-life or life-like contexts, raises an additional dimension to the problem, and necessitates consideration of the role of field and context in argumentation

> Many methods of didactic education assume a separation between knowing and doing, treating knowledge as an integral, self-sufficient substance, theoretically independent of the situation in which it is learned and used. The primary concern of schools often seems to be the transfer of this substance, which comprises abstract,

> decontextualized formal concepts. The activity and context in which learning takes place are thus regarded as merely ancillary to learning….Any method that tries to teach abstract concepts independently of authentic situations overlooks the way understanding is developed through continued, situated use. … A concept for example, will continually evolve with each new occasion of use. (J. S. Brown et al., 1989, pp. 32-33)

‘Field’ and ‘context’ in argumentation play two vastly different, but analogous roles, and they serve to place parameters on the argument and the argumentation practice itself. The term ‘field’ was used by Toulmin (1958) to distinguish the community purpose or setting in which the argument is employed. For example, political argument for the purpose of electioneering is largely rhetoric which employs specific persuasive devices that rarely have a platform for immediate response by the audience: similar to advertising campaigns of any nature. A legal argument to be presented to a jury, while similarly persuasive in that it seeks to promote certain views and obfuscate others, is usually prepared with the expectation of challenge and rebuttal. Scientific argumentation is often intended to further knowledge, as such rebuttals are both expected and required to further understanding and provide new, contestable conjectures or explanations for phenomena. In the case of science, persuasive devices would be inappropriate within the relevant community.

‘Context’ is more specific to the problem itself and serves to guide the interpretation of any response. “To solve an ill-defined problem … whatever it takes to close its open constraints must be sought out or generated by the problem solver himself” (Reitman, 1965, p. 164) and this will be strongly influenced by the context of the problem. To illustrate, the question “What is an acceptable rate of parameter non-compliance in manufacturing?” may have a completely different answer if plumbing valves are being manufactured rather than replacement heart valves.

> One unavoidable consequence … is that no solution to an ill-defined problem can count on universal acceptance. Any such problem by definition involves open constraints. … It may well turn out that settings of these open constraints acceptable to one individual are unacceptable to the other. Consequently, solutions involving these settings also may not be acceptable. (Reitman, 1965, p. 153)

However, these may be acceptable under some conditions, constraints or assumptions and not under others. Thus the context of the problem becomes an essential and inextricable component of the problem itself as the context influences and determines the acceptability of assumptions.

Argumentation as such is not the focus of this research, but rather argumentation in school mathematics. Thus it becomes essential to consider the context in which the argument is embedded. Students can answer a seemingly mathematical problem quite effectively using non-mathematical grounds. While their answers might be considered valid and even effective or persuasive outside of the field of mathematics, the intent in the classroom is to develop a response which is field-dependent on mathematics, and which addresses the context. Approaches that embed learning in activity and make deliberate use of the social and physical context are more in line with the understanding of learning and cognition that is emerging from research (J. S. Brown et al., 1989, p. 32).

Ill-structured Problems

The second key component to IBL problems is that they are ill-structured. To fully understand an ill-structured or ill-defined problem, it is perhaps easiest to consider what the contrasting well-structured problem is. Minsky (1961, p. 9) describes problems he terms *well-defined* as those in which there is a *systematic way to decide when a proposed solution is acceptable.* Reitman (1965) describes well-structured problems in terms of the initial states and goal states being firmly defined in the problem statement. One example used by Reitman (1965) is that of a jigsaw puzzle. With a jigsaw puzzle, the initial state is clear, the end state can be determined and assessed as being correct (acceptable), or otherwise. The solution method may vary; some people like to complete the edge pieces first while others will focus on pictorial aspects of the puzzle such as completing an object of one colour first. However, it is clear when the end state is reached and there is little to challenge.

Reitman (1965) argues that the majority of energy that people expend in problem solving is in engaging with questions that do not meet Minsky's criteria for well-defined. These problems do not pre-define "necessary and sufficient conditions for a solution" (Reitman, 1965, p. 148) and that it is difficult to even determine what it means to have solved a problem that is ill-defined. Simon's (1973) characteristics of ill-structured problems include those which have no definite criterion to judge a solution; no mechanisable process to apply the criterion; the problem space is not meaningfully defined; boundaries can be breached by new alternatives; interactions (for example, with a real context) can alter constraints. Reitman (1965) stresses that problems are not simply ill-structured or well-defined but rather exist somewhere on a continuum; further, there will be variation within the problem in that some points may be highly constrained while ill-defined at other points.

In the traditional mathematics classroom, didactic approaches are employed with the expert knowledge deriving from the teacher or textbook. Virtually all textbook problems could be situated towards the extreme end of well-defined on Reitman's (1965) continuum. Worked samples are provided, practice examples undertaken and then corrected according to answers provided in the text or teacher resource materials. Against Simon's (1973) proposed characteristics of problems, it is easy to see that this constitutes a well-defined problem: definite criteria for success, a mechanistic process, a meaningfully defined problem space, a lack of alternatives, and no 'muddying' by interaction with a context exists.

Accepting Reitman's (1965) contention that the majority of problems faced in life outside the classroom are ill-defined, it would follow that students of mathematics also need exposure to ill-structured problems which require the application of mathematics to solve. This is not intended to undermine the teaching of pure mathematics any more than it is intended to undermine the need for spelling, punctuation or grammar in writing. However, all students will, in day to day life, need to solve problems with minimal constraints to determine a best-case solution; just as the student of writing needs to compose text as well as learning the mechanics. To illustrate, consider the problem, "What would be the best value floor covering to use in the new rumpus room?". The problem is clearly ambiguous; what is meant by 'best'? The most durable, practical, cheapest, longest-wearing, easiest to clean, most suitable for pets, warmest, coolest, most environmentally responsible, or asthma and allergy friendly? The constraints may include budget, availability, preferences, and room purpose. The possibilities are virtually endless and there does not exist "necessary and sufficient conditions for a solution" (Reitman, 1965, p. 148), only a best-case scenario given the constraints determined by the problem solver at that time.

In the flooring example above, it is easily conceivable that new problems would arise during the addressing of the original problem. These new problems are such that their solution may assist in the solving of the original problem: for example, determining a shared understanding of the nature of activities to take place in the rumpus room. Under many circumstances, both the ultimate solution and the intermediate steps along the way, need to be explained, defended and justified.

The discussion of problems in terms of structure is an important one: for Reitman (1965) and Simon (1973), it was largely because they were looking at problem solving methods using artificial intelligence. In inquiry, this distinction is just as important. While well-defined problems offer some scope for inquiry approaches into solution method, ill-defined problems may not have a solution, or may have multiple solutions and multiple possible pathways that can be identified and negotiated. Furthermore, small changes in the problem will often lead to significant changes in the solution (Eraut, 1994, p. 45). As a result, students need to then be able to explain, justify and defend their choices, decisions and outcomes using supporting evidence; a practice variously described as ‘explanation and argumentation’, ‘argumentation’, or ‘knowledge building’ by various researchers and to varying degrees of specificity (Berland & Reiser, 2009). Thus, IBL provides an ideal classroom environment for adopting an argument-based pedagogy.

Another reason for delineating problem type is that a significant amount of the research undertaken into mathematics argumentation focuses on developing a mathematical proof or a theorem, the very problem that Reitman (1965) provides as an example of a “typical, well-defined problem” (p. 149). While the problem solver may select from various, though limited operators to solve the proof, the initial state and the goal are determined. In many respects, a similar format is taken to informal proofs in mathematical classrooms. Students may be posed with a problem 54+99, which may be calculated using any of several methods of varying efficiency and elegancy; however, the end state is determined as 153. In such a situation, the problem solver is left to defend and justify only the pathway taken to reach the solution, not also the solution or even initial state of itself, and the assumptions or choices made in the process. Thus, the ill-structured nature of inquiry questions also provides the ambiguity necessary for argumentation in mathematics.

Accordingly, the nature of the task, or inquiry problem, is as essential consideration in the planning and conduct of the teaching and learning activities.

The Community of Learners

The third key component to IBL is the addressing of problems within a community of learners. Essentially, a community of learners is distinguished by a culture in which all participants are involved in a collective effort to create or build knowledge (Brown & Campione, 1996). It is through the creation of collective knowledge that the knowledge of the individual is developed, and this is promoted through communal sharing of understandings. An underlying premise is that not every student must have the same knowledge: rather students can develop different understandings that come together to contribute to a whole body of knowledge. This is contrary to the goals and culture normally associated with schooling, which takes more of an individual focus, “discourage[ing] the sharing of knowledge, - by inhibiting students from talking, working on problems or projects together, and sharing or discussing their ideas” (Collins, 2006, p.55). Bransford, Brown, & Cocking argue that:

> Teachers must attend to designing classroom activities and helping students organize their work in ways that promote the kind of intellectual camaraderie and the attitudes toward learning that build a sense of community. In such a community, students might help one another solve problems by building on each other’s knowledge, asking questions to clarify explanations, and suggesting avenues that would move the group toward its goal (Brown and Campione, 1994). Both cooperation in problem solving (Evans, 1989; Newstead and Evans, 1995) and argumentation (Goldman, 1994; Habermas, 1990; Kuhn, 1991; Moshman, 1995a, 1995b; Salmon and Zeitz, 1995; Youniss and Damon, 1992) among students in such an intellectual community enhance cognitive development. (2000, p. 25)

IBL problems are well suited to being addressed within a community of learners as the ill-structured, ambiguous nature of the inquiry question (Anderson, 2002) lends itself to multiple approaches and pathways. These questions need to be negotiated in order to decide on an approach, and groups of students may well address problems in different ways and develop different knowledge or responses to such questions. It is only when the groups come together to discuss their progress that the ‘pieces’ come together to develop broader understanding.

3.6.2 Familiarity with content and context

The nature of IBL is that it is set within a context, and this is a significant part of its appeal for use with argumentation as the context is necessary to generate a focus for the argument. However, the context needs to be such that it fosters both a level of engagement with the students, provides the potential for a deeply mathematical approach, and enables connections to be made that demonstrate its usefulness.

The processes of learning argumentation skills are of themselves complex; to incorporate them in an unfamiliar, disengaging, or highly sophisticated context could only serve to deeply frustrate students. Conversely, there is some evidence that if students are highly concerned with an issue, they may have trouble decentring from their perspective (Douaire, in Muller Mirza et al., 2009, original in French)**.** Ideally, a context is needed that will provide or sustain students’ interest level necessary to generate sufficient student commitment to the argument, yet not be so emotive as to make rational approaches difficult. Fielding-Wells and Makar (2008b) identified five characteristics of contexts which were found to engage students more highly in IBL: contexts of high interest; those students could relate to it in terms of real-life or personal experience; those perceived to be of high value; those which enabled students to experience enjoyment; or, those that were novel or challenging. While all five aspects were not reported present in one problem context, the presence of these aspects individually was suggested as serving to motivate students.

The second consideration would be to ensure that the context supports the goal of deepened mathematical understanding: the problem should be able to employ mathematics in order to develop a solution. As there is little research, if any, into argumentation in ill-structured, ambiguous mathematical learning contexts, this is an envisaged ideal, rather than a researched finding. Equally able to be envisaged is the potential for the context to take a dominant position over the mathematics, such that it obfuscates the mathematics, if it is too exciting or emotive. It could also be suggested that the context should be real or familiar to students if the goal is development of mathematics concepts. An unfamiliar context could essentially make the application of the mathematics more difficult than no context at all, or would require significant work to be completed to have students at a level of understanding which acts in a beneficial way. Of course, if the context is one which is embedded in the curriculum, the approach could serve to build both mathematical understandings and cross-curricular content.

The final point is that the context should be authentic. Research reports students see limited application of mathematics and question its usefulness (McPhan et al., 2008); thus we should establish a real reason for wanting to find a solution. Ideally, the context could also reflect some part of the curriculum so as to draw on topics which are certain to be familiar to all students, regardless of background experiences, and can also reinforce cross-curricular learning and utility of mathematics outside of the mathematics classroom.

The context may actually serve a far more significant role than would be anticipated: to the extent that the student's relationship with argument context may provide one explanation for research findings which are contradictory in terms of the argumentation skills that young students are able to demonstrate:

> This apparent disagreement can be resolved by noting the roles of two factors, the tasks and two types of knowledge, knowledge of subject matter and knowledge of argument-related verbal structures or schema. Young children have experience in conflict situations and they become personally engaged in them. They have encountered peer and parent-child interpersonal conflict. When they enter into argumentation, their knowledge and experience in social relationships is activated along with their related argument structures…whether or not a person is able to perform reasonably in an argumentative situation depends upon context, which includes the argument's contents. (Voss & Van Dyke, 2001, p. 102)

3.6.3 Need to explicitly teach argumentation

Argumentation is not typically taught in Australian schools, with the single exception of high school science. Related genres taught in primary English are often the persuasive letter and the formal debate, with the latter more frequently prepared as notes and presented in an oral format. However, the focus on these forms of writing is on developing a 'winning' position with the audience, as distinct from reaching a defensible solution. In both cases, a skilled orator can present a case which may be emotive and appealing, but lack logical justification or defensibility. This may be an acceptable position in certain junctures but not in developing a mathematically or scientifically robust response to a real life problem.

Pontecorvo and Pirchio (2000) have conducted considerable work with young people in this area and argue that argumentation is not a naturally occurring process, that there are social conditions and socialising experiences necessary for development. These skills include talking and listening, knowing the meaning of argument, positioning, justifying with evidence, constructing arguments, evaluating arguments, counter arguing/debating, and reflecting on the argument process (S. Simon & Richardson, 2009). Providing students with arguments alone to discuss is not sufficient to develop the skills or processes of valid argumentation (Osborne et al., 2004), rather argumentation is a form of discourse that needs to be explicitly taught through suitable instruction, task structuring and modelling (Jimenez-Aleixandre & Erduran, 2007).

3.6.4 Argumentation as a socially supported pedagogy

"Argumentation, then, is not simply a matter of constructing a logically coherent series of statements, it is a socially situated event designed by the arguer to satisfy the demands of a particular context" (Berland & Forte, 2010, p. 428). Argumentation involves consideration of differing views of a topic in order to obtain the best possible resolution or outcome. Essentially then, argument involves engagement in dialogic processes with another, even if the 'other' is in fact oneself presenting a contradictory viewpoint: "This washing powder is cheaper but this one cleans better, or maybe I should try that biodegradable one while it is on special, after all it is better for the environment". As such, argumentation necessitates the community engage in social intercourse for multiple and varied purposes: negotiation, collaboration, and presentation (the latter including justification and defence).

In addressing ill-structured and ambiguous problems, there is a necessity to engage in a significant number of interrelated sub-problems from which a few must be chosen as "the search through all possibilities will be too inefficient for practical use" (Minsky, 1961, p. 9). Thus, decisions must be made about; the purposes of the inquiry, what is meant by the question, what parameters to adopt to narrow the question for practicable purposes, and which of a multitude of potential approaches would be useful. "This decision must be based on 1) estimates of relative difficulties and 2) estimates of centrality of the different candidates for attention." (Minsky, 1961, p. 21). This clearly requires that students engage in negotiation around the topic, putting forth ideas, defending those ideas and engaging openly with the ideas of others, with the aim of developing an efficient approach to addressing a meaningful problem.

Student to student interaction is an essential part of the development of argumentation skills with the importance of collaboration unable to be overestimated (Jiménez-Aleixandre et al, 2000; Zohar & Nemet, 2002). Collaborative work strongly supports the development of necessary discursive and reasoning skills in students, with evidence that learning is enhanced when children share their understandings, challenge each other's ideas, evaluate evidence, and consider various options in a reasoned manner.

The third interaction type is that of presentation. The student who is presenting an argument, whether more formally as an address, or informally as an opinion within a group, benefits from the presence of others. Research findings have suggested that students experience more difficulty in developing the written form of an argument due to the absence of interlocutors. In the written form of an argument, the author is responsible for the establishment, examination and expression of multiple viewpoints; as distinct from oral argumentation where the presence of one or more interlocutors serves to fulfil that function. "In argumentative dialogue, the presence of two individuals face to face seems to act as a support for the child in understanding the other person's point of view and adapting to it" (Muller Mirza et al., 2009, p. 72).

While the argument for student interaction in argumentation is strong, there needs to be consideration given to young students or novices engaging in argumentation as they may well be unaccustomed to determining and considering alternate perspectives. So while, engaging in argumentative discourse with others is a practical step in envisioning and developing different perspectives, engaging with peers alone in these instances would be less productive. Hence it becomes the teachers' role to teach, develop and model argumentation skills. As the children interact socially with others, including adults, they will learn to modify and construct their understandings of their world, language use in general, and argumentative discourses in particular (Muller Mirza et al., 2009).

In summary, “argumentation, then, is not simply a matter of constructing a logically coherent series of statements, it is a socially situated event designed by the arguer to satisfy the demands of a particular context” (Berland & Forte, 2010, p. 428).

3.7 Student Specific Factors

A final consideration, after addressing the nature of the classroom and the learning activities, is that of the students themselves. While students may observe and engage in argumentation practices informally from a young age, there are developmental considerations that may well impact on their readiness to engage in classroom settings. These considerations are addressed below.

3.7.1 Developmental readiness

While as young as 2 or 3 years of age, children demonstrate argumentation in a day-to-day context, this argumentative discourse is still highly undeveloped (Muller Mirza et al., 2009). Research from the field of psychology suggests that specific processes of argumentation do not develop until children are significantly older. As was observed previously, argumentation skills are usually developed naturally by children as part of their social interaction initially with family or carers (Pontecorvo & Arcidiacono, in Muller Mirza et al., 2009, original in French) and later with teachers and peers. In the very early years, children will engage in persuasion through non-verbal forms of communication, such as aggressive gestures or crying, or verbal forms, such as threats, in attempts to persuade others; by around the age of three children demonstrate the production of justifications (Muller Mirza et al., 2009). Around the age of 7, children can assume a position and provide a defence for it but do not offer any significant counter-argument or response to being challenged (Stein & Miller, 1993).

Piaget suggests that, at this same age, the child begins to separate self from world; emerging from an egocentric perception of their own view as central to the universe. The child is then able to look beyond his or her own perspective to consideration of the points-of-view of others. Until the child develops the ability to decentrate “he will not spontaneously seek to convince others, nor to accept common truths, nor, above all, to prove or test his opinions” as he “supposes that everyone necessarily thinks like himself” (Piaget, 1960, p. 33). It is perhaps the word spontaneously here which should draw attention. Astington (1994) discusses empirical studies that indicate children as young as

3, 4 and 5 years of age have demonstrated decentration in contexts meaningful to the children. This raises the question as to what is possible in terms of argumentation in younger students if appropriate supports, structures and tasks are employed.

3.7.2 Recognition of a need for evidence

When children make an assertion they tend not to see a need for evidence to support that assertion (Fielding-Wells, 2010; Muller Mirza et al., 2009). Piaget suggests that the use of evidence or justification again makes its appearance at around age 7, tying in with the diminishing egocentricity of the child and recognition that, with other, perhaps opposing viewpoints in play, they may need to defend their position. Systematic justification is only seen at around 13-14 years of age (Golder & Coirier, 1994): a similar age to Piaget's Formal Operational Stage at which point he suggests students would be capable of defending their viewpoint and considering the viewpoint of others simultaneously (Piaget, 1960).

This is a particularly important consideration as the notion of 'developing good arguments' is one which needs careful consideration in the mathematical and scientific fields. To create a 'good' argument to a student may mean providing a biased version of events; a situation which is not acceptable in scientific practice. For example, Stygall (1987) observed students being selective about evidence they chose to report and even making deliberate decisions to not report data which contradicted their claims, relying only on that which provided support. Whereas, Fielding-Wells (2010) noted that students often made impulsive and instinctive claims in response to ill-structured questions without paying appropriate attention to evidence. Neither of these approaches is acceptable in the disciplines of science and mathematics.

3.8 **Research Questions**

The literature outlined above suggests the need for further research of argumentation practices in inquiry-based learning in primary mathematics. The proposed study is designed to build on these current understandings and at the same time address significant gaps in the literature.

In particular, many of the contributions identified above regarding argumentation and inquiry-based learning have come from research in science classrooms. The nature of science learning has many practical and procedural differences from the practice of mathematics. It cannot be assumed that the lessons learned from the science context are directly transferrable to the mathematics classroom. Given the relative recency of the introduction of Inquiry-Based Learning (IBL), as defined here, as a pedagogical approach into mathematics, there has been very little research conducted into argumentation in IBL. Argumentation research that has occurred within the discipline of mathematics is largely that associated with mathematical proof (see, for example, Conner, 2007; Forman et al., 1998; Giannakoulias, Mastorides, Potari, & Zachariades, 2010). Accordingly, there is both an absence of documented research knowledge in this field and little conceptual understanding of the potential argumentation might have for mathematics learning in the primary years.

In addition, there is no assurance, by simply introducing argument to a classroom, that students will acquire or develop any or all of the potential benefits discussed previously. There must be a “coordinated, complex and systematic set of pedagogical, curricular and assessment initiatives” that are specifically engineered to develop and extend students’ adoption of argumentation practices (Jimenez-Aleixandre & Erduran, 2007, p. 5). Currently, there is little conception of what initiatives may be introduced, how they could be introduced effectively, when they might be appropriate in terms of the curriculum, or what level of explanation and persuasion may be considered usual, ideal or acceptable for younger students.

As a result of the literature and identified gaps in literature, the aim of the proposed study is to develop a broad theoretical understanding of argumentation and the development of argumentative practices in the primary mathematics classroom. In particular, the following questions are intended to be addressed:

1. What are key features of an Inquiry-Based Argument model as implemented in a primary (elementary) mathematics setting?
2. What signature elements of Inquiry-Based Argument can serve to guide children’s mathematical argumentation?

3.9 **Conclusion**

Argument and argumentation processes are constructs with multiple meanings and purposes in both common usage and within more formalised studies of discursive practice. In this instance, an epistemic argumentation focus (Biro & Siegel, 1992; Lumer, 2010; Siegel & Biro, 1997) has been adopted in order to deepen students connections with mathematics as evidence and to adopt a Knowledge Building approach to seeking a best possible outcome as distinct from a winning position. Potential benefits to introducing argumentation into mathematics are extensive if previous research in the field of science education is indicative. However, implementation of argumentation practices into mathematics education cannot be guaranteed to have similar benefits. Accordingly, there is a need to direct research towards the potential for argumentation, as well as the practicalities of implementation in primary mathematics teaching and learning. The next chapter will address a selection of functional and structural approaches to argumentation, and provide justification for those adopted as a theoretical framing for this research. Means of assessing argument are also addressed as it is essential to have a means of ‘measuring’ development in argument quality when working with the students.

4 Theoretical Framework

4.1 Chapter Overview

This purpose of the first half of this chapter is to provide a theoretical framing to the research. In order to do so, argumentation is first separated into two approaches, both of which are essential considerations in argumentative practices: structural analysis of the argument; that is, the actual components that make up an argument, and functional purpose or goal of the argument. This first half of this chapter addresses both of these approaches as adopted for this research. The second half of the chapter addresses the nature of argument assessment; both structurally and functionally, covering issues that surround determining whether students' arguments are progressing.

Structural approaches, such as that of Toulmin (1958), focus heavily on the component parts of an argument and the form and role of these elements. The use and identification of the elements of an argument offer opportunities for analysis of the argument in terms of components and their linkages. For example, it is possible to state that grounds have not been provided or that an argument is unwarranted. Functional approaches to argumentation focus rather on the goals and purposes of argument, for example, Berland and Reiser's goals of argumentation (2009), and van Eemeren et al.'s (1996) forms of argument. These approaches offer a progressive model of argumentative discourse by looking at the intent of the discourse and the nature of the wider discursive practice in which the argument is established. The functional approach may take into consideration the nature and role of audience, the purpose of the argument and the selection of the grounds. These approaches are elaborated in this chapter, along with the associated considerations for assessing argument quality, in order to provide the framework used for the implementation and analysis of the research.

4.2 Structural Approaches to Argument

Argument can be considered from multiple perspectives; for example, Toulmin (Toulmin, 1958; Toulmin et al., 1984) considered argument in terms of structure: decomposing the argument into its component parts and examining those components both individually and in terms of their interactions. Toulmin's structure enables us to closely examine the claim, grounds, warrants and so forth, to determine and critically evaluate the soundness of an argument. The structural approach of Toulmin will first be addressed as it provides the language with which to address other aspects of argumentation. This will be followed by a consideration of criticisms of the model and a look at an alternate, simple model that has been built from the work of Toulmin but simplified to enable more appropriate structure and terminology for working with children.

4.2.1 Toulmin's model

Toulmin et al.'s (1984) classical work in this field provides both a clear structure of argument and a framework to enable consideration to be given to the assessment of an argument's soundness, strength, and any use of fallacious assertions. In order to provide clear and common understanding in terms of the soundness of an argument, Toulmin and his colleagues (1984) detailed four elements that compose an argument: claims, grounds, backings and warrants. The claim is the initial assertion that identifies the destination of the "argumentor", explaining the stance and position taken. The grounds serve to provide the underlying support that is required to enable the claim to be accepted as reliable and valid. It is the information or understanding (primary evidence) that the claim is based upon and which leads to the claim or assertion being made. Warrants and rules enable the checking of the grounds, to determine whether they offer genuine support for the claim or whether the grounds are irrelevant or unwarranted. These provide the justification for moving from grounds to claim. Warrants may take the form of laws of nature, legal principles and statutes, rules of thumb, or engineering formulas, for example. However, a warrant is not self-supporting: there exists a necessity to explain or justify the warrant, to indicate the reliability and applicability of the rule, law, formula, or principle that is being relied upon, and to indicate the reason for the argumentor's confidence in such a warrant. This support for the warrant is termed the backing and it provides a validation of the use of the warrant. For example, the backing may indicate that scientific laws have been heavily supported, research findings have been checked and so forth. An example of the elements of an argument is illustrated in Figure 4.1. This example uses the components of an argument that could be advanced in response to a question posed to students in the research addressed in this dissertation. However, arguments are often not as simple as the example depicts and consist of multiple sets of grounds, warrants and backings to support a claim.

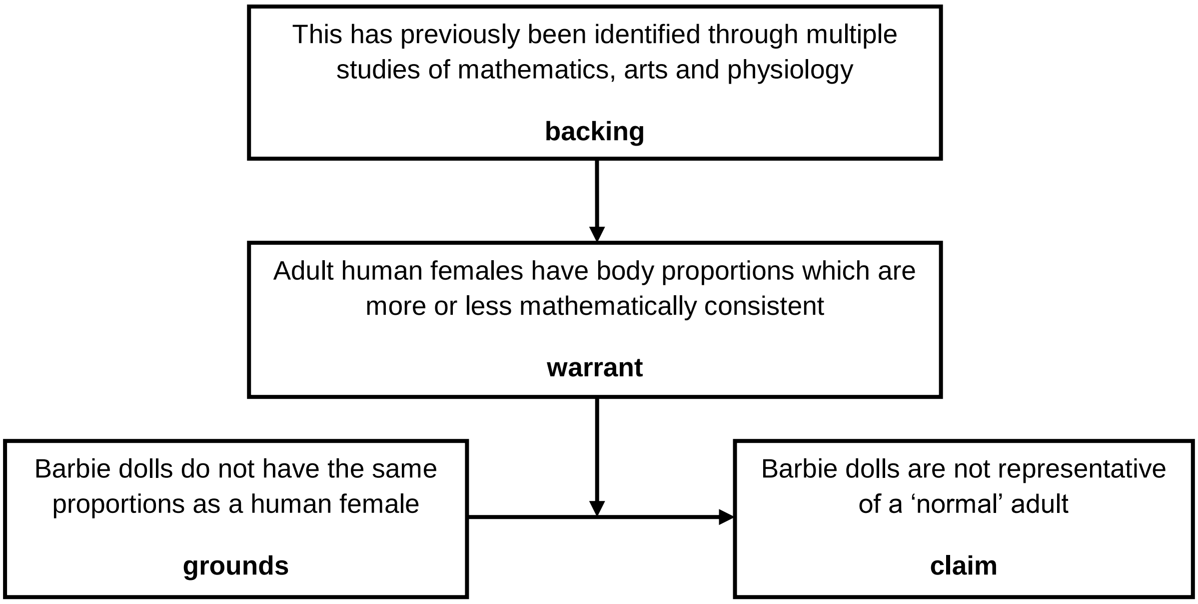


Figure 4.1: Simple representation of Toulmin’s elements of an argument

This use of the elements of an argument enables the soundness of an argument to be analysed in terms of its components and their linkages. It makes it possible to determine whether the requisite parts of the arguments are present. For example, where a claim is made with no supporting grounds, the claim is arbitrary and lacking in soundness. Addressing soundness is not the same as determining the strength of the argument. In terms of strength, Toulmin et al. suggest that

> only within the abstract arguments of pure mathematics can our statements be linked together by relations of “absolute necessity”: in all practical realms, the connections that we have to deal with are more or less qualified, and more or less conditional. (1984, p. 81)

In practical situations then, it is usual that conclusions, or claims, are based on ‘imperfect’ evidence, requiring determination of the strength of the argument along something of a continuum rather than an all or nothing scenario. Thus, we may consider the necessity to qualify arguments.

In Toulmin’s structure, qualifiers take the form of probabilistic language, indicating the strength between the grounds and the claim. For example, “David is the only player not scheduled to play a tennis match this Saturday, so *presumably* he will be the coach’s first choice for a fill-in”. The use of qualifiers may further lead to the expression of a rebuttal or exception; an acknowledgement of a weakness to the underlying claims. For example, “David is the only player not scheduled to play a tennis match this Saturday, so presumably he will be the coach’s first choice for a fill-in *unless Josh has recovered from his injury”.* Figure 4.2 demonstrates this schematically, using our example from before. This language also enables the expression of a tentative discovery to be posited and the identification of presumptions.

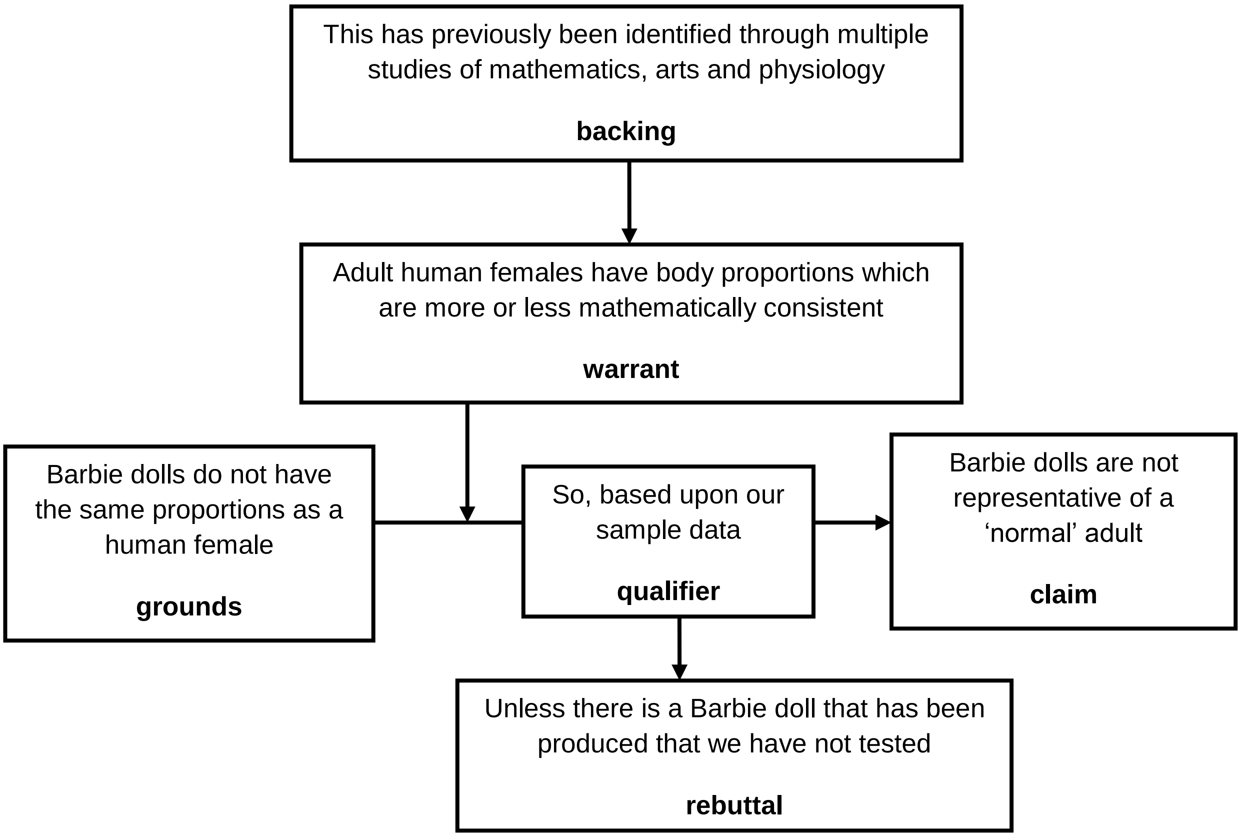


Figure 4.2: Representation of Toulmin’s elements of an argument incorporating qualifiers and rebuttals

Toulmin’s model is not without criticism, some of which is related to its use in analysis of arguments and other which is related to the omission of contextual features. Some of the pertinent issues are addressed below.

4.2.2 Criticisms of Toulmin’s model

Toulmin (Toulmin, 1958; Toulmin et al., 1984) detailed a structural model that aimed to address all arguments, regardless of context. As such, the proposed structure was generic and was neither designed to account for all aspects of an argument, nor a range of argument purposes, including affective and contextual elements of an argument. In addition, the technical focus on structure does not support consideration of the nature of the evidence, warrants and backing provided, but rather only whether they are provided. Criticisms of Toulmin's model have been made along several lines; most of which relate to its practical, rather than theoretical usage.

As an approach to practical argumentation, Toulmin's model "does not account for the non-logical aspects of arguments, such as affective and stylistic elements, that are integral and essential aspects, as well as opportunities for persuasion" (Schroeder, 1997, p. 97). Toulmin was primarily concerned with rationality when in practice, rhetoric can be persuasive without being logical or based in fact. As an example, many an argument has been advanced by a superior, or in a diplomatic setting, which appears overtly accepted due to the high importance of the relationship and perhaps a much lower importance of disputing the argument itself, or a strategic decision to 'pick the battles'.

> Toulmin claims to be considering a context, which is actually a limited context consisting of the argument itself and its immediate surroundings, and ignores or excludes the wider context in which the actual negotiations of power transpire…ideological choices are camouflaged by separating the arguments from the social and political arenas from which they emerge. (Schroeder, 1997, p. 103)

Failure to account for a power imbalance could have significance in school contexts, to the extent that the cultural context of the classroom may need to address the difference in power between teacher and students, and within the student cohort: imbalances that may derive from social interactions or perceived capabilities. The discounting of context also serves to make the decomposition of an argument problematic and increases the difficulty of distinguishing between data, claim, warrant (Erduran, 2007). Kelly, Druker, and Chen (1998, p. 856) argue that these components need to be contextualized within a conversation in order to be adequately identified and considered in relation to other comments and linguistic cues.

A second difficulty of a predominantly structural approach is that while an argument may be correctly structured with its entire component parts, there is no guarantee that the evidence presented, nor the reasoning, is epistemically acceptable. Kelly et al. (1998) stress the need for consideration of multiple types of data, and varying degrees of acceptability, *depending on the context/field of the argument* [emphasis added]. In the case of mathematics, we have a science that is characterized by an epistemology of valuing fact and logic over the emotive. As young students have a propensity to respond intuitively and affectively when engaged in mathematical and statistical inquiry problems (Muller Mirza et al., 2009), this leaning towards fact and logic is essential in this context.

A third consideration is that the Toulmin model exhibits a strong reliance on a hierarchical or linear approach to knowledge and knowing: the model necessitates a claim which is supported by various elements. “The Toulmin model … forces the analyst to determine the claim and then how other material in the discourse is utilized to develop and support that claim.” (Kneupper, 1978, p. 240). While “Toulmin’s conceptual and visual models … imply a lateral form of thinking, [they] misrepresent the inherent hierarchical patterns in his approach” (Schroeder, 1997, p. 104). Sanborn advocates dismissing a purely hierarchical approach as restricting ways of thinking and knowing in a significant portion of learners. Rather, she promotes the inclusion of a web-logic approach to expository writing that takes a less linear focus, arguing not “all good expository writing follow[s] the dictates of the thesis-driven, hierarchical essay” (Sanborn, 1992, p. 144).

In short,

> Toulmin’s system is not a system of logic but rather an elaborate system of justification. Despite demonstrating the interrelationship of the various elements of an argument, Toulmin’s system … does not provide a means for evaluation… Toulmin’s system of argumentation provides, often in retrospect, the foundation for a claim but guarantees neither the validity of the claim nor the soundness of the argument. (Schroeder, 1997, p. 100)

In conclusion, while the argumentation structure may focus students on the necessity of making a claim and providing evidence, it does not provide a means for teachers and students to evaluate the logic or strength of their claim. Secondly, a simpler model than that proposed by Toulmin would appear to be indicated, such as the Claim-Evidence-Reasoning model devised by McNeill and associates (McNeill & Martin, 2011; Zembal-Saul, McNeill, & Hershberger, 2013). This enables a more general focus on the primary components of classroom argument. However, it is important to acknowledge the Toulmin model as the more sophisticated underpinning model as this potentially provides the necessary discourse to extend students' knowledge and the underlying principles that are addressed in the CER model.

4.2.3 CER model

The CER model derives from the more complex Toulmin model of argument but has been adapted to be suitable as a framework for scientific education (McNeill & Krajcik, 2008, 2011, 2012; Zembal-Saul et al., 2013). This model is represented diagrammatically in Figure 4.3 (McNeill & Krajcik, 2011; McNeill & Martin, 2011; Zembal-Saul et al., 2013). The claim and evidence components align with Toulmin's claim and grounds: *claim* being the conclusion that addresses the original question and *evidence* being the scientific data that supports the claim. In classroom science practice, this data may come from an investigation that students complete or from observations, reading material, archived data, or other sources of information (Zembal-Saul et al., 2013). In explaining their model, Zembal-Saul et al. (2013) maintain that the data needs to be both appropriate and sufficient to support the claim.

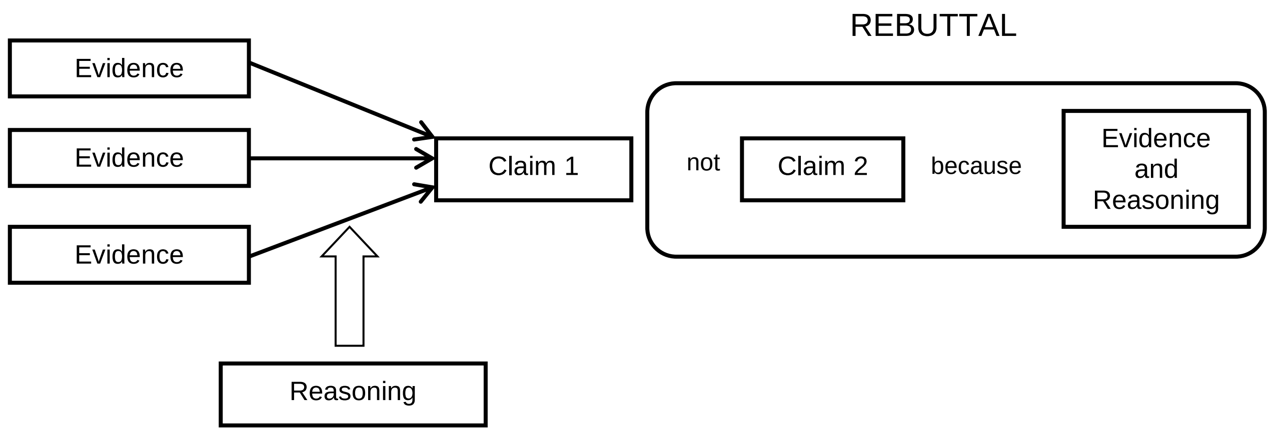


Figure 4.3: CER Model (reproduced from McNeill and Krajcik (2011))

The third component, *reasoning*, encompasses the warrants and backing; or the logic that enables the grounds to be used to establish the claim (McNeill & Krajcik, 2012). In science education,

> the reasoning should include the big science idea or science concept that is the focus of the lesson. Including the reasoning encourages students to consider and reflect on these science ideas, as well as provid[ing] them with the opportunity to

> become more comfortable using scientific terms and language. (Zembal-Saul et al., 2013, p. 25)

Thus, reasoning is the justification that shows why the data count as evidence to support the claim and should include appropriate scientific principles. The reasoning brings in the scientific background knowledge or scientific theory that justifies making the claim and choosing the appropriate evidence. McNeill's model (Zembal-Saul et al., 2013) also incorporates rebuttal, or the process of examining and discounting counter-claims.

As can be seen from this model, there appears to be opportunity to address some of the criticisms of Toulmin's work, with the potential to apply web-logic responded to through the incorporation of multiple forms of evidence, and a more simplistic model that does not necessitate the identification of grounds, warrants and backing as separate identities. However, not all aspects of the limitations of a structural-only approach can be addressed by remodelling Toulmin, for example, the relationships inherent in argument and aspects of context and purpose. So while Toulmin and McNeill focus on argument structure, others, such as van Eemeren et al. (1996) focus on the various forms argument can take; analytic, rhetoric and dialectic, which largely centre on the relationships between the interlocutor and audience. Berland and Reiser (2009) differ again by considering the purpose of the argument; whether it is to understand, to explain or to persuade. Each of these perspectives is contributory to the entire construct but applies a different lens and thus will now be considered.

4.3 Functional Approaches to Argument

Berland and Reiser (2009) propose three hierarchical levels of explanation and argumentation—understanding, explanation, and persuasion—and these levels are largely determined by the respective goals of sense-making, articulation, and persuasion. The purpose of sense-making is for students to develop a personal understanding of the phenomena under investigation. Evidence is at the core of the sense-making and this sense-making must rest on an alignment between evidence and claims (Driver et al., 2000). Sense-making consists of an inwardly focused belief system and therefore may lack a reliable, valid evidence-base or stem from an incomplete and unchallenged position. Through the lack of challenge, personal understandings that emerge from sense-making may not have been adequately called into contest and may therefore be based on incorrect premises, flawed logic, misconceptions, or erroneous constructs. Publicly articulated understandings are, by nature, available to be critiqued and questioned whereas unarticulated understandings are not open to the scrutiny of others and therefore may remain unchallenged.

At the second level, the goal of the argument is articulation; that is, for the proponent to construct and explain to an audience their reasoning and make evident their claim. This level of argumentation is potentially much deeper. Putting students in a position that requires them to ‘go public’ situates them in such a way that they need to more closely consider their position and supporting reasons, and their need to make their links clear, thus engaging more deeply and critically with the evidence they put forward (Berland & Reiser, 2009). The primary goal of this level is to develop initial shared understandings of the phenomena under study.

The third level of argumentation is persuasion. It differs from articulation in that while the goal of articulation is explanatory the goal here is to convince; to develop the most robust explanation of the studied phenomena. As the community of learners puts forth their views and evidence, students are required not only to articulate their findings and claims, but also to be able to defend, justify and reflect (Berland & Reiser, 2009). This necessitates a deeper understanding of the phenomena and the evidence offered in its support.

As can be seen, this model addresses the relationship between the proponent and audience, whether the audience is self or public, and also the influence of interaction with the audience. A second critical aspect here is the role of evidence. The evidence an individual accepts for the purpose of understanding may well be less rigorous than that which is offered with the intent to persuade; thereby enabling a focus to be placed on the quality of evidence. Thus, this model addresses many of the aspects that were not incorporated into the structural approaches.

Parallels to Berland and Reiser’s goals of argumentation can be made with van Eemeren et al.’s (1996) proposed a model of forms of argument. Again three levels are specified and based around the discursive practices of argumentation, these are; analytic, rhetoric and dialectic. However, differences are significant enough to warrant addressing this model separately.

The *analytic* phase involves intrapersonal communication only: similar to the sense-making stage, the proponent endeavours to develop their own understandings and make sense of the evidence themselves. The second phase is *rhetorical*, which necessitates the use of one-way communication of the argument and the supporting evidence in an attempt to convince others. This is one of the more significant differences to Berland and Reiser's (2009) goals of argumentation, as their goal for one-way communication was explanation and articulation rather than an attempt to convince or persuade. The significance is that rhetoric does not necessitate the use of evidence (Duschl, 2007). Aristotle (1954) provided the definition of rhetoric as the "faculty of observing in any given case the available means of persuasion". In Aristotelian terms, rhetoric is the study of oration; it extends to discovery of what is persuasive in varying circumstances, including positioning and emotiveness. In more modern parlance, rhetoric can be considered a form of one-way communication that may only seek to explain, to advance evidence. Alternatively it may be manipulative and employ persuasive tactics and fallacy. As such, while rhetoric is considered an art form in some disciplines, its role in advancing scientific and mathematical knowledge is questionable unless the definition of rhetoric as explanation is employed.

The third phase, *dialectical*, or interactive communication, is where the audience is responsive and able to challenge, argue and dispute evidence, claim and reasoning. These stages progressively necessitate deeper consideration of the evidentiary links as the degree of potential challenge increases (van Eemeren et al., 1996).

Consideration of both van Eemeren et al's *Forms of Argument* (van Eemeren & Grootendorst, 1992; van Eemeren et al., 1996) and Berland and Reiser's *Goals of Argument* (2009) enabled a tentative hypothetical model to be developed to address the role of evidence in these argument forms. Based on the literature, several initial hypotheses for the proposed study were incorporated into a model (Figure 4.4) to predict possible implications for evidence as it is used by students. Essentially, the model firstly identifies a loose relationship between the goals and forms of argument.

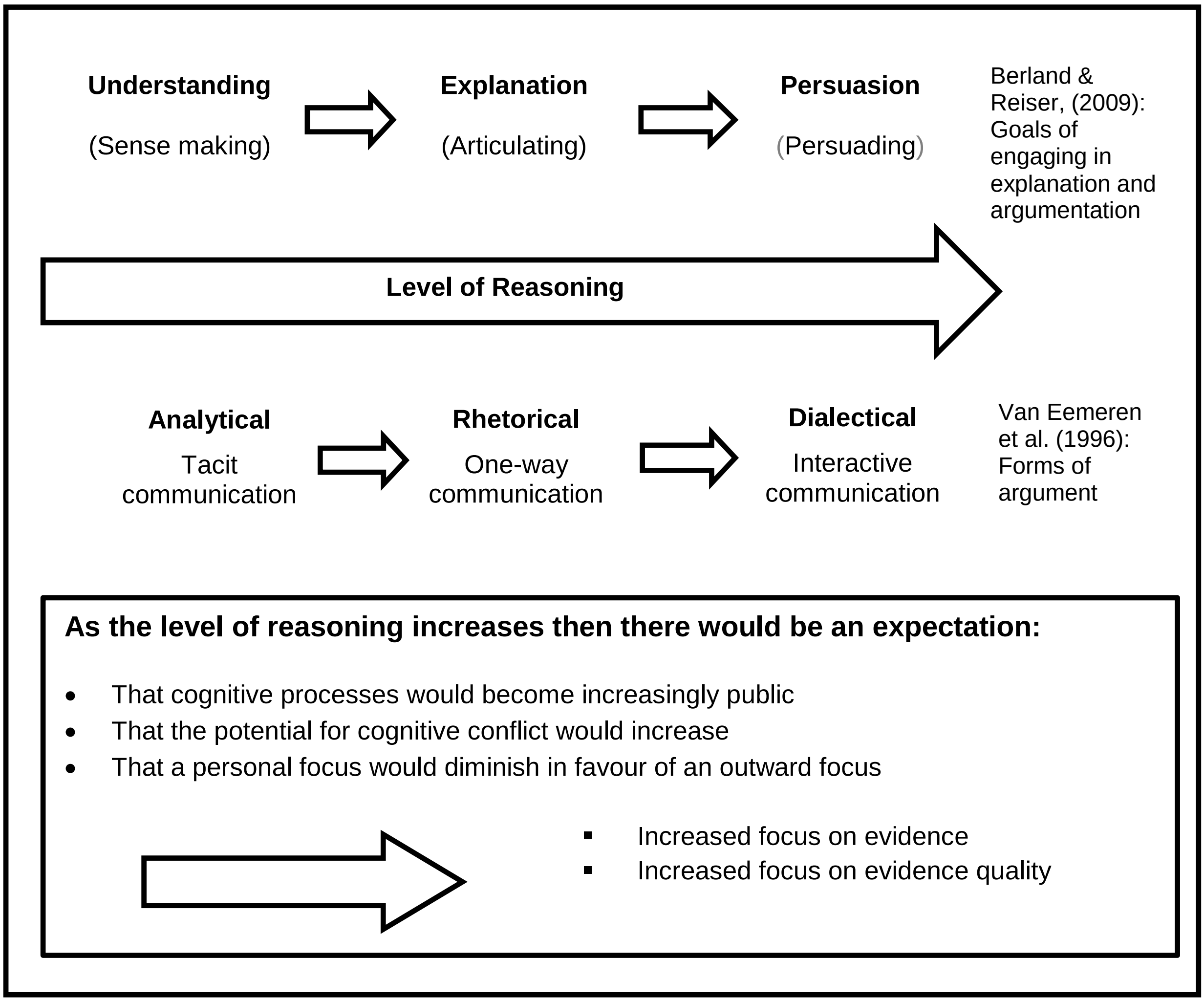


Figure 4.4: Model indicating potential interactions between reasoning, forms and goals of argumentation and use of evidence.

While students are engaged in Understanding and Sense- Making about phenomena, their dialogue is largely self-focussed and internalised. They may be thinking about evidence, trying to understand what that evidence means but are possibly not yet at a stage where they can articulate their developing ideas, or perhaps don’t have call to. Regardless, those ideas and any evidence the students have, may remain internalised and oriented to self. If, through the practices of argumentation, students are positioned to explain or articulate their understandings and their evidence (if any) for those understandings, then there is potential for several occurrences. The first is that the teacher and the other students have opportunities to access the thoughts of the articulating student. In such a way, the student’s cognitive processes become public and the teacher is better positioned to appreciate any understandings that require challenging. The second is that, as the student becomes more familiar with discussing ideas, they may begin to anticipate having an audience, and thus become more accustomed to organising ideas and evidence in order to present them effectively. A third outcome may be that, as the students listen to the ideas and understandings of others, they begin to become more aware of contrasting and conflicting ideas, some that may be equally or better supported than their own. Thus they take a more outward focus, reflecting on others' ideas rather than dismissing them as not agreeing with their own. In such instances, the consideration of the evidence and the strength of the evidence become a focus for analysing and evaluating the strength of disparate understandings.

Finally, as students are positioned to present claim, evidence and reasoning in a forum in which it can be discussed and disputed, it would be anticipated that the potential for cognitive conflict would reach its highest as other students are no longer simply reflecting on ideas that may be different to theirs, but are being openly challenged to justify and defend these ideas. This would be anticipated to have a significant effect on both the provision of evidence and on what constitutes quality evidence, as students search for potential flaws in the collection, interpretation and quality of the evidence presented. It is here that the epistemic nature of what constitutes knowledge within the discipline of mathematics becomes a focus. Other factors that may impact on evidence include the level of scrutiny; whether the audience is expert, peer or novice in terms of both context and experience; potential outcome, whether high-stakes or low stakes; classroom culture in terms of support for risk-taking; and personal relevance and interest in the topic under study.

In order to ascertain whether there is change in the nature of evidence and the evidence quality as students engage in argumentation practices, there needs to be a mechanism for identification of such. There are many criteria established for the assessment of argumentation (see, for example, Sandoval & Millwood, 2005; Toulmin et al., 1984; Zohar & Nemet, 2002) and these vary between structural and functional methods. Consideration of several approaches likely to be of use in the context of research into mathematical argumentation has been provided below.

4.4 **Assessment of Argument – Toulmin's Structural Approach**

The purpose of teaching and using argumentation in the classroom is not to develop better arguments for their own sake, but rather to develop a stronger focus on mathematical evidence and reasoning. Therefore, in terms of assessing the argument, the structure is not the focus but rather the epistemological acceptability of the components of the argument and the logic which connects them. However, we need to recognise the structural components of the argument in order to identify absence of such as the most fundamental level of inadequacy: the absence or lack of supported claim.

Toulmin et al. (1984) raise three questions of central concern and which surround the structure of the argument itself:

- How are claims to be supported by reasons? (logicality of claims)
- How those reasons themselves are to be evaluated? (validity, relevance, strength, linkage)
- What makes some arguments, such as trains of reasoning, better and others worse?

In order to scaffold students into developing strong, valid arguments, a thorough understanding of how arguments fail is desirable. Toulmin et al. describes breakdown in the grounds or warrants as 'fallacy' and offers a model identifying five broad types of fallacy; with fallacies being deemed "arguments that can seem persuasive despite being unsound" (Toulmin et al., 1984, p. 132). These categories have been identified as potential organisers for evaluating the structure of student arguments in order to assist in the determination of common fallacious thinking. These are as follows:

- <u>Fallacies that result from missing grounds</u>: Claims for which no real evidence is produced.
- <u>Fallacies that result from irrelevant grounds</u>: Claims for which the evidence does not pertain directly to the claim.
- <u>Fallacies that result from defective grounds</u>: Claims for which the evidence produced is insufficient.
- <u>Fallacies that result from unwarranted assumptions</u>: Claims which presume that the grounds lead to the claim when in fact they do not. This is often a result of

the inappropriate acceptance of an assumption regarding the applicability of a warrant.

- Fallacies that result from ambiguities in the arguments: Occurring when some term in the arguments can be construed in more than one way.

The worth of Toulmin's analysis lies in its ability to be used to deconstruct arguments to their component parts and in this way enable a focussed appraisal of each aspect of an argument to weigh the validity of that component. A claim may be supported by reason; however this only identifies it as an argument in terms of structure. It does not in any way validate the reasoning or give standards by which that reasoning can be assessed. We do not need to be familiar with either the field the argument comes from, or the epistemological criteria for reasoning within that field, in order to identify the essential components of an argument. Essentially "we recognise reasons and claims, not because of their logical function in argumentation, but rather because of their communicative function in discourse" (Bermejo-Luque, 2006, p. 76). However, in order to assess the validity of the claim-based reasoning, knowledge of the field is required. Hence it could be considered that the argument must also be assessed in terms of field-dependence or context, and according to the epistemology of the field.

4.5 Assessment of Argument – Functional Approaches

Functional approaches to argument assessment centre on evaluation of the reasoning or logical nature of the argument more so than on the structural components. Two approaches in particular have been identified as being of potential use and address the quality of arguments and the identification of arguments that adopt evidentiary fallaciousness (as distinct from Toulmin's structural fallaciousness).

Sampson and Clarke (2006, pp. 659-660) proposed five criteria for examining the quality of student's scientific arguments in general terms and provided a structure for teachers to use. These criteria and questions that serve to identify the guiding focus of the criteria are provided in Table 4.1. While Sampson and Clarke identify these as a first step towards examining scientific arguments, the criteria they suggest focus on the nature of knowledge within the discipline of which the argument is a focus, rather than simply the structure of the argument itself. These criteria then offer potential for framing and guiding the evaluation of argumentation in mathematics learning, through a similar focus on what constitutes acceptable argumentation in mathematics.

Table 4.1: Criteria for examining the quality of scientific arguments

Criteria	Assessment Focus
Criteria 1: Examine the nature and quality of the claim	• Does the claim address the inquiry question asked? • Does the claim co-ordinate with available evidence? • Is the claim scientifically accurate?
Criteria 2: Examine how far the claim is justified	• Was evidence provided to justify the claim? • Was it the right kind of evidence? What is being relied on as evidence? (for example empirical data or personal experience/opinion)
Criteria 3: Examine if the claim accounts for all the available evidence	• Was there a focus on data patterns or on single pieces of evidence to support own beliefs? • Is all available evidence considered, including anomalous or contradictory evidence?
Criteria 4: Examine how the argument attempts to discount alternatives	• Were other plausible arguments considered? • Was there an attempt to deal with them by providing any potential weaknesses?
Criteria 5: Examine how epistemological references are used to coordinate claims and evidence	• How was data gathered and interpreted? • Was design or methodology considered when evaluating the evidence? • Were these activities done in accordance with community standards?

A further model that enables the assessment of arguments within science is that of Zeidler (1997). Zeidler addresses argument fallacy in a way that differs from Toulmin (Toulmin et al., 1984), in that Toulmin's fallacies are largely centred on the structural components and technicalities of the argument itself: the provision of evidence, the extent to which the evidence is related to the claim and so forth, but not the extent to which the evidence is epistemically acceptable. Zeidler by contrast, centres argument fallacy on the nature of the evidence and the acceptability of the evidence; for example, the collection and analysis of the evidence. Rather than pinning these fallacies on a theoretical model, Zeidler's categories derive from synthesized research studies into classroom argumentation in schools with students: his purpose being to ensure that teachers were aware of these fallacies in order to benefit their practice. The five categories Zeidler identifies as problematic are:

- Problems with validity – affirming a claim because the proponent felt it to be true rather than based on evidence.
- A naïve conception of argument structure – selecting evidence according to the proponent's bias and discount oppositional data.
- The effects of core beliefs on argumentation – lacking of examination of counter-evidence and criticism that contrast with the proponent's core beliefs.
- Inadequate sampling of evidence – failing to recognise too little evidence.
- Altering the representation of argument and evidence – use of additional assertions or inferences beyond the data/evidence available.

These identified fallacies are compatible with the indicators of quality argument provided by Sampson and Clark (2008) above, lending strength to the essential nature of evidence in the evaluation of scientific arguments. These fallacies, and the structure of Toulmin's argument, provide a framework that enables deep and systematic analysis of students' arguments. Together with the conceptual and hypothesised relationship between Berland and Reiser's (2009) goals of argument and van Eemeren et al.'s (1996) forms of argument (Figure 4.4), the use of these frameworks is anticipated to assist by providing a lens with which to look at students' mathematical arguments.

The chapter following will provide a methodological framework for this research that will serve to provide the reader with an understanding of the approaches taken in the research, including the details of the research design, the situational context, the interventions implemented and finally the sources and analysis of data. The methodological design draws on the research provided in the literature chapter and serves to explain how the theoretical framework led to design decisions made and planned analysis.

5 Methodology

5.1 Chapter Overview

This chapter provides the background necessary to contextually situate the research methodology. In order to do this, the details and justification of the decision to adopt Design Research (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003; Lesh, 2002) are addressed, along with details of how the principles of Design Research were incorporated into this study. Details of the research site, participants, and the teaching and learning interventions are described in sufficient detail, not to replicate the research, but to provide adequate background that the results chapters and conclusions drawn are able to be evaluated by the reader. Finally, the method of data analysis is detailed with reasoning for the adopted approaches provided.

5.2 Research Aims and Research Questions

The aim of this exploratory research was to develop pedagogical theory of inquiry-based argumentation in mathematics. In particular, the following questions were addressed:

1. What are key features of an Inquiry-Based Argument model as implemented in a primary (elementary) mathematics setting?
2. What Signature Elements of Inquiry-Based Argument can serve to guide children's mathematical argumentation?

5.3 Research Design

This research sought to develop pedagogical theory through multiple iterations of reflective-prospective cycles of improvement. Accordingly, Design Research was chosen as a methodological stance (Cobb et al., 2003; Lesh, 2002). This is because Design Research essentially entails engineering forms of learning and then systematically studying the learning within its context, which was ideal for the research purpose. Design Research is characterised through the following five features (Cobb et al., 2003):

- **Theory creation:** The research purpose is to develop theories about the process of learning and the designed support or scaffolding of that learning.
- **Innovative Intervention:** The nature of the research is highly interventionist and the intent is to investigate the possibilities for educational improvement; as such, the researcher is not constrained from making improvements or alterations to successive learning materials.

- **Reflective and Prospective Processes:** The researcher creates the conditions for developing the theories but also takes a reflective approach to challenge those theories.
- **Iteration:** An iterative approach is adopted with successive cycles of generating conjecture, developing methods and ideas, testing them, and reflecting critically on the progress and process before generating further ideas and theories.
- **Pragmatic Stance:** The work is by nature pragmatic; it must have some use beyond a philosophical orientation. That is, the work aims to specify learning processes involved and result in the development of some practical application. The results are therefore humble and specific to the context.

In this study, these features are built into the design in the following specific ways (Table 5.1).

Table 5.1: Implementation of features of Design Research to the proposed study.

Design Research Feature	Implementation / Alignment to Project
Theory creation	The aim of this exploratory research was to develop pedagogical theory of inquiry-based argumentation in mathematics.
Innovative intervention	An intervention was designed which enabled an experienced mathematical inquiry teacher to engage with conceptual and pedagogical tools to foster and study students' argumentation practices.
Reflective and prospective processes	Two levels of reflective-prospective practice were identified. At the macro level, current theories were examined, and potential frameworks identified which served to guide the argumentation process.
	On the micro level, the overall progress of the class was monitored on a lesson-by-lesson basis to reflect upon progress and determine how to proceed: each lesson was responsive to the previous lesson and the students' progression.
Iteration	The study incorporated repeated cycles of intervention, analysis and reflection, which in turn identified a focus and guided the development of the next cycle of learning. Two unit cycles were planned for the class. Each unit was also cyclical in nature to enable the research-practitioner to plan, implement, respond and assess lessons as an ongoing means of developing learning sequences and focus.
Pragmatic Stance	The research was intended to develop understandings about the nature of open-ended argumentation which would lead to practical applications for classroom use.

Design experiment is built on two unique premises about the research purpose and its context that set it apart from other methodological designs (Lesh, 2002). The first is the focus that design research gives to making the research relevant to practice. In design research, this premise is enacted by the inclusion of practitioners in the research design, the implementation, and the interpretation of results. That is, while there is a distinction between the role of the practitioner and the researcher, a collegiate approach is taken so that both, for example, may be engaged in envisioning, planning and analysis.. Teachers constantly interact with students, teaching materials, and content understandings on a daily basis. In developing successive views of all of these, they constantly refine their opinions, recognising new needs, opportunities and issues that arise. Acknowledging this insight and experience capitalises on their participation as co-researchers and valuable sources of data and evidence.

Lesh's second premise underlying the context for design research studies is that the classroom is a complex, evolving, dynamic system that cannot be objectively observed, where learning and teaching are intertwined with a host of other factors that are social, emotional and cognitive in nature. The researcher creates interventions to the system which affect the nature of this intertwining and therefore creates a subjective system (Lesh, 2002). Furthermore, the active selection of what to teach, what to observe, what to record and analyse, how to code and what to report are subjective of themselves as they highlight certain aspects of the research and, by definition, obscure others. This is not seen as a weakness of the intended design however, as the construct of interest is exceedingly complex and unlikely to be addressed effectively using linear methods.

The aim of this research study is to contribute an empirical foundation for a pedagogical theory that develops argumentation practices in inquiry-based primary mathematics classrooms. This type of learning differs significantly from current educational practices and therefore conditions must be explicitly created in which the instructional theory can be developed and tested.

5.4 **Context**

5.4.1 Background

The research study reported in this dissertation originated from the participation of the author in an initial investigative research project (Makar, 2011) into teacher development of IBL in primary mathematics classrooms ('IBL research project'). The author was one of the original teacher participants in the IBL research project and became deeply interested in the development of children's cognitive engagement during IBL. Specifically she noted that children initially responded to inquiry questions with virtually instantaneous, intuitive responses. However, these 'answers' were usually unsupported and often demonstrated anecdotal and unsystematic reasoning (Fielding-Wells, 2010). Students could be assisted through the inquiry process to arrive at supported answers through the collection, organization and analysis of evidence and the subsequent offering of a conclusion. This process was fraught with difficulties for the students; however, it appeared that these very difficulties were the source of the deepest engagement and learning for the students. Therefore simply directing students through these activities was thought to be counter-productive to the purpose of inquiry and the students' developing reasoning and thinking skills.

At the time this research was undertaken, the author was a teacher at the school and the findings here are from extended work with the author's own class (author as a research—practitioner). The author had held a teaching connection with the school for around seven years. Thus the students were familiar with the author in her role as a teacher and the author was established in the school setting. The research was explained to the students and their role as co-researchers was co-opted at the outset.

5.4.2 Research site

The school in which the research was conducted is a metropolitan government primary school in Queensland, Australia. This school is a relatively large primary school with approximately four drafts of each year level from Preparatory through to Year 7. The school had/s an average Index of Community Socio-Economic Advantage (ICSEA), a significant proportion of international students from predominantly Asian nations, and a low Australian Indigenous population. The school has a Special Education Program for students with Autistic Spectrum Disorder and it is usual to have approximately four students in a class with varying degrees of high-functioning autism. The class engaged in the research was a ‘rolling’ class and remained with the same teaching team across both Year 4 and Year 5.

At the commencement of the research, the school site had been part of the IBL research project for seven years, involving a number of teachers at the school. Due to the nature of the site, with some teachers using IBL and others not, students had been exposed to a range of teaching over the course of several years: some were quite familiar with learning through inquiry and whereas others had little or no experience, and a range of possible combinations in between.

5.4.3 Participants

Classes at the Prep, Year 1, Year 3, Year 4 and Year 5 level were initially involved in this doctoral research project. However, the results presented and discussed herein describe the development of one class, the author’s own, as Year 4’s and then the following school year as Year 5’s. This decision was made in order to develop and present a deeper, richer illustration of the students’ developing reasoning. Of the classes engaged in the project, this class was chosen to report on as the students had significant prior experience with inquiry and thus more easily moved into an argumentative focus with little preparation. Research from the IBA project that has not been presented here has been reported elsewhere (Fielding-Wells, 2014; Fielding-Wells & Makar, 2012, 2013).

The subject class was taught under a shared teaching partnership by two teachers with six years each of IBL experience and a keen interest in inquiry methodology. One teacher had over 30 years of experience as a primary teacher: the other eight years. These teachers taught the same class for two consecutive years, enabling an uncommon opportunity for deepening both inquiry practice and opportunities to develop argument structure and practice. In particular, the researcher was one of the teachers of this class, which enabled her to use it as a testing ground of tentative and emerging theoretical and practical ideas.

5.5 **Planned – Enacted Intervention**

The research consisted of cycles of preparation and design, experimental teaching, and then analysis and reflection, which in turn led to the next teaching cycle. These cycles of preparation-teaching-reflection occurred within two broad and distinctive units, each of which is described in some detail below.

5.5.1 Overview of the learning and teaching sequence

Two units were implemented in order to address the intended learning and teaching sequences. These units are overviewed in Table 5.2. Each inquiry unit was developed around an inquiry question as the driving focus for obtaining evidence and making a claim. The intended mathematical content and the argumentation focus are also identified in this table. Following the table, an explanation of the structure of the units, followed by an overview of the units themselves, is provided in greater detail.

Table 5.2: Overview of Learning Sequence

	Research Question (Context)	Mathematical Content Knowledge	Argumentation Structures	Argumentation Process
1	Does Barbie have the same proportions as a human?	Proportional reasoning Informal representation Fractional representations Informal statistical inference Distribution Central tendency Samples vs populations Data representations – tallies, dot plots	Informal introduction of Claim – Evidence Role of Evidence	Informal introduction of Claim – Evidence links Challenging evidence
2	Can a pyramid have a scalene face?	Geometrical reasoning Properties of triangles Properties of pyramids Angles	Formal introduction of claim, evidence, reasoning and qualification Quality of evidence Scaffolded argument	Envisaging and gathering evidence Mathematical Reasoning

5.5.2 Unit planning overview

The unit series above were designed according to a model previously developed by the author and her colleagues (Allmond, Wells, & Makar, 2010). Allmond et al. identified four phases of mathematical inquiry which they coined Discover, Devise, Develop and Defend: these are described briefly below. It should be noted that these phases, while generally sequential, may involve backtracking to test and adopt new approaches as required.

The Discover phase reflects the need to engage students with the context of the teaching sequence. The purpose is threefold: to act as a hook with which to engage students; to immerse students in the context; and, to ascertain students' prior experiences, interests and knowledge of the context.

The Devise phase engages students in negotiating the inquiry question and the meaning of the question; planning how they will approach the question, and envisaging the evidence that will be needed to answer the question. The students then conduct their plan, and collect, record and analyse their evidence in the Develop phase. It is common in this phase for students to encounter difficulties they did not foresee with their plans and to make adjustments as they proceed.

Finally, in the Defend phase, the students respond to the initial question with an answer (or claim), present their evidence, and communicate and justify their solutions. Some background on the students' experiences prior to the commencement of the units has been described below.

5.5.3 Background

Prior to the commencement of the research intervention, students had been briefly introduced to the Evidence Model: a model which had served well in previous experiences of IBL (Fielding-Wells, 2010) and which is depicted in Figure 5.1. This model serves to focus students on the components of an Inquiry. 'Purpose' identifies the wider context which is often ambiguous and ill-structured. The purpose is refined to a researchable question for which the students plan how to obtain evidence, gather evidence, and then form a conclusion which addresses the question asked. This class had some existing familiarity with IBL in Mathematics, as they had been previously and briefly introduced to the model; however, the author did not feel the students had more than a rudimentary appreciation and were yet to do more than identify the terminology. For example, while the students were able to identify that evidence was needed, they had no experience of visualising what evidence might be needed or what might constitute acceptable evidence either in terms of the context or mathematics.

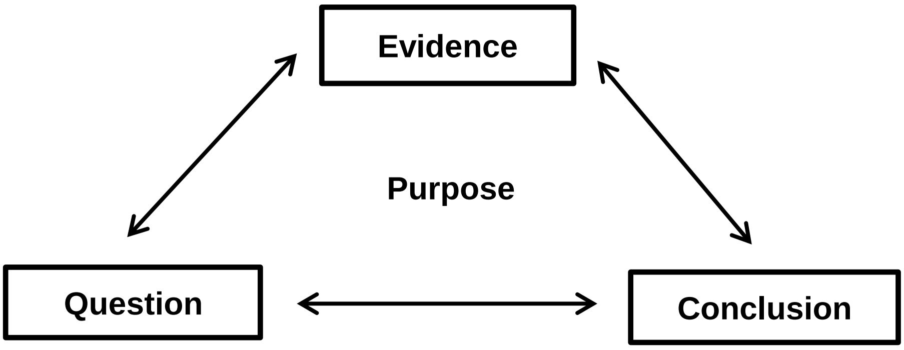


Figure 5.1: Evidence Model

Research literature indicates that in the early stages of scientific argumentation, children tend to make impulsive, intuitive assertions without seeing a need for evidence to support them (Jiménez-Aleixandre et al., 2000) and so it was anticipated that similar responses may be noted in mathematical argumentation. Accordingly, the first unit incorporated a strong focus on the need for mathematical evidence and the use of mathematical evidence to support the claim. The focus of the second unit was enabling deepened student reliance on evidence, extending skills in explanation and argumentation, and focussing students on determining and developing quality evidence.

Between the development of the first and second units, the school made an administrative level decision to shift away from an integrated curriculum. Hence the first unit had an integrated context, with links outside the curriculum, whereas the second unit was embedded in a stand-alone mathematical context. An additional difference was in the constraints of planning; that, while the school administration and year-level teaching cohort had significant input into the first unit, the second unit had fewer constraints and the teacher was free to plan work independently of the other year-level teachers. This enabled planning to be responsive to student interests.

In both instances, summative assessment was incorporated into the design. While Knowledge Building takes a greater focus on the collective development of the class, institutional necessities of reporting students' progress dictated the requirement for individual, summative assessment.

5.5.4 Unit 1: Developing recognition of the need for mathematical evidence – *Is Barbie a monster?*

Rationale

The first unit was collaboratively designed by two of the Year 4 teachers (not involving the author) and the school's Head of Curriculum. This unit incorporated assessment tasks which required the design of a dress reflective of the style of Valentino (an exhibition of Valentino's designs was on display at the Gallery of Modern Art and was to be visited by the students), with an artist's statement justifying the selection and use of artistic elements (line, tone, colour, shape etc). These dresses were to be designed, constructed and modelled on a Barbie doll (for an overview of Barbie refer to, for example, Wikipedia, 2013a). As the Head of Curriculum was challenging more teachers to include a mathematical focus into their integrated units, the author was approached to adapt this aspect only. The author's intent was to develop opportunities for taking an IBA focus, and thus proposed the question "*Could Barbie be a human?"* with the intent to guide the students to refine the problem to consideration of proportion. The question was amended by the original planning team to "*Is Barbie a Monster*?" as they thought this would appeal more to boys. As the integrated unit was enacted, three year level teachers taught the concepts of proportion through direct teaching. However, the unit was amended for research purposes to take an IBA focus, thus enabling aspects of argumentation to be addressed by the class reported here. These amendments included the use of the question as an open-ended, ambiguous Inquiry question to enable the students to debate the question and refine it to something able to be addressed mathematically (Allmond & Makar, 2010) and for which evidence could be sought and claims made. Prior to this lesson, the students had undertaken several units of IBL in mathematics in which the students had become familiar with the Evidence Model (Figure 5.1) and it was intended to capitalise on this model to focus students on the relationship between Question, Evidence, Conclusion and Purpose (Fielding-Wells, 2010). While the intended focus of the unit was presented in Table 5.2, the enactment of the unit sequence was largely responsive to the students; hence the overview provided below is retrospective of the enacted progression of the units. Unless otherwise indicated throughout the dissertation, 'teacher' refers to the author. A brief overview of the phases of the unit of learning is discussed below.

Inquiry Phases

Discover Phase: 2 hours

In the initial phase, the students were introduced to the question "Is Barbie a monster?", and encouraged to explore how the question might be addressed. This was done to engage student interest and introduce the idea of proportion while making links to Visual Art work in which students had previously engaged with facial proportion. The students negotiated and refined the question to "Could Barbie be human?" and then "Does Barbie have human proportions?": thus making a shift to a question that could be addressed mathematically. A discussion ensued around the Evidence Model to develop recollection of the need for evidence.

Devise Phase: 2 hours

In the Devise phase, student groups were tasked to make plans for how they might obtain evidence to address the question. Students then presented their initial plan to the whole class in order for the teacher to elicit and address the difficulties students had, and to ensure that all groups had a plan and had a way to obtain evidence. The groups collectively struggled with this, so in order to provide direction, students were asked to individually write a conclusion and support it with imaginary evidence. Having engaged with this idea, students were able to contribute to a class discussion based on the points that arose from their conclusions. The teacher was able to use this discussion to encourage students with accurate conceptions and more articulately formed ideas to share with the class and thus refine their own understandings. Responses the students offered were challenged and a deeper focus on proportion, as distinct from an absolute length, was achieved (for greater discussion on the development of students' proportional reasoning in this unit, refer to Fielding-Wells, Dole, & Makar, 2014). The students determined in discussion that they would need a range of adult human proportions to contrast against Barbie and consideration was given to which measures would be appropriate for the purpose. The teacher used questioning to bring the students to the realisation that they had insufficient knowledge of proportion and so modelling using concrete materials and representations was directly taught. This was an important step as the students needed to have the mathematical underpinnings to progress with the unit but the teacher wanted them to see the need and to envisage proportion first.

Develop Phase: 5 hours

Students were directly taught, initially using modelling and then the algorithm, to calculate human proportions from the measurements they had obtained. Students expressed these ratios as a single number when recording; for example, a ratio of 1:1.6 was recorded simply as “1.6”. The students were encouraged to use calculators as they had not yet learnt how to divide numbers of this size, and to round the answers to tenths for simplicity. Data from a single proportion (length of hand: length of face) was displayed for the students to see prior to their being led to visualise the data by considering the scores and developing a sense of what the scores would mean in terms of proportion; that is, what might ‘normal’ proportion be limited to? (Could a person have a 1:0.1 ratio for hand to face for example?). This enabled the teacher to address outlying scores and have students consider what they meant in context – strengthening the more abstract concept of ratio.

In their initial ‘imagined’ claims, students had represented their data in a multitude of predominantly disorganised ways, if at all. Discussion around possibilities for representing data had the students considering the merits of organised lists, dot plots and tallies: the purpose being for them to consider the importance of representations in interpreting evidence and explaining that evidence to an audience. Each student was assigned a different proportion (eg height: arm span) and the student was challenged to determine what was ‘normal’ for a human for that proportion. Students were not expected to share these data, it was just to get a sense of ‘normal’ for themselves in order to establish a sense-making position (Berland & Reiser, 2009), and this was made explicit to the students. A discussion held with the students assisted them to realise that having a response for one proportion was not sufficient, that students would need to share their answers with each other. It was suggested then that the students could put their response onto a poster to provide the other students with responses and evidence. In this way, the students would be situated in a position of explanation (Berland & Reiser, 2009) and a comparison to sense-making responses could be made. The blinded posters were shared and students asked to provide feedback as to what aspects were helpful to their understanding and make suggestion for improvement. Collectively the class then discussed their findings to establish their own criteria for effective explanations.

Defend Phase: 6 hours

Students had an opportunity to reflect on their newly created criteria and on the specific and individual feedback they each had from their peers, before creating a final poster that would be convincing to their classmates. Students then established Barbie's ratio for the proportion they were working with and made comparisons to their established human range of 'normal' scores. A sample data set was displayed by the teacher and discussion ensued as to how a decision could be made as to whether Barbie could fit into the range of humans or not. Through discussions, students observed the shape of the distribution of human scores and began to make claims based on what they had determined was 'normal' for a human. This information was added to the students' posters. Each student made a presentation to the class during which their explicit goal was to convince the class of their data range for what was considered normal, and their decision as to whether Barbie fit into that normal range, was justified. The students were explicitly aware that they could be challenged by the audience. Each student then presented their findings to the class, supported by their claim and evidence poster. This provided students with the opportunity to persuade others, thus the greatest opportunity to have their responses challenged and to be required to defend it (Berland & Reiser, 2009).

Summative Assessment

Finally, after each student had presented and been engaged in challenging the evidence and claims of others, students were provided with a fresh set of data for a proportion they had not worked with (and one in which Barbie purposely lay outside of, but close to, the main cluster of data scores. The reasoning used to justify the call as to whether Barbie could be human was of significant interest as it provided good insight into student understandings. The data set also included an extreme outlier and a score close to the main clump to observe students' data handling and reasoning for decisions.

5.5.5 Unit 2: Developing explanation and argumentation - *Can a pyramid have a scalene face?*

Rationale

The second unit was planned around a focus inquiry question posed by a student while working within a non-inquiry based unit on geometry: "Can a pyramid have a scalene f?" [Note: the students tended to use the term face and side interchangeably in the inquiry until they began to appreciate that the correct term was ‘face’]. As the question was student instigated, it was decided that students would likely be sufficiently interested in the content to make this topic suitable for a second IBA unit. The argumentation focus planned was to deepen students’ reliance on evidence while developing their skills in explanation and argumentation. To do so, the students were introduced formally to the Claim-Evidence-Reasoning (CER) Model (McNeill & Martin, 2011; Zembal-Saul, McNeill, & Hershberger, 2013), along with engaging in a focus on developing and critiquing quality evidence. The opportunity to engage students in a non-externally contextualised topic was also thought to potentially provide contrasting insight into argumentation in mathematics. Furthermore, the content coverage had the potential to deepen what were essentially surface understandings the students had of the attributes of 3D shapes (specifically pyramids).

As before, the unit has been written retrospectively as it was largely responsive to student needs, and cycles of refection and prospective processes (Cobb et al., 2003). Again the curriculum descriptors are located as an appendix (Appendix B) and a brief unit overview, as enacted, is provided below.

<u>Inquiry Phases</u>

Discover Phase: 2 hour

Initially, students engaged in discussion to review the existing Inquiry model (Question-Evidence-Conclusion) and to address what students thought constituted quality evidence. The need for a Claim and Evidence were reviewed and Reasoning was added to assist the students to see the need for a justification from Evidence to Claim.

Devise Phase: 3 hour

The class was encouraged, in small groups, to envisage the evidence they would require to enable progress with the question. This information was then shared as a whole class, with groups explaining the plans they had for providing useful evidence. This served to establish a wide range of models and representations which enabled the students to see that multiple forms of evidence were possible and to provide avenues for continuing with their inquiry. During this class discussion, the need for providing students with a means for limiting their claims became opportune and so qualifiers were introduced to enable students to express parameters and variations to their claim.

Develop Phase: 3 hours

Students engaged in evidence gathering through their individual group approaches: building models, nets and other representations to address the problem. Ongoing discussion in groups, facilitated by the teacher through questions, was used to deepen mathematical understanding and develop the use of mathematical, particularly geometrical, vocabulary. The students were afforded autonomy; however complex issues which affected students were brought to class discussion for whole-class resolution. This was done to enable the class to address group problems collectively in order to further develop the class as a knowledge-building community. Issues addressed by the students included the length of sides which were adjacent on the pyramids and the sizes of the angles on the faces. As students shared their progress, they were encouraged to challenge developing evidence in order to encourage improved quality and reliance on accurate constructions of representations.

To have students focus on each component, and to provide a scaffold for producing the components of an argument, the students were provided with a series of ordered questions/guidelines:

- What claim are you making?
- What grounds do you have for making this claim?
- What is your reasoning?
- Could you convince someone else?
- How sure are you?

Defend Phase: 3 hours

The groups presented their interim findings to the whole class to provide an opportunity for students to challenge each other to identify any issues, naïve conceptions, or inaccuracies in evidence or reasoning, and again improve evidence quality. These challenges ensured that the students were able to identify and demonstrate the attributes of a scalene triangle and a pyramid, and provided guidance for students to refine representations. At the completion of the presentations, the teacher also worked to encourage students to look for patterns, generalisations, and hypotheses they could test in order to have students collectively consider all of the group’s findings and to see the importance of collaborative knowledge-sharing. Through this process, students were encouraged to put forth hypotheses in order to build knowledge collectively (Bereiter & Scardamalia, 1996).

<u>Summative Assessment</u>

Individually students provided a written claim, evidence (own choice of representation/s), reasoning and, if required, qualifier(s). The assessment focus was the demonstrated understanding of argumentation process and product, and the knowledge and representation of the associated geometric concepts.

5.5.6 Philosophical underpinnings of the unit design

Prior to conducting the research, and based on a broad understanding of the literature, a tentative impression of context knowledge and content knowledge (mathematical in this instance) coming together to feed into the development of argumentation was envisaged (Figure 5.2).

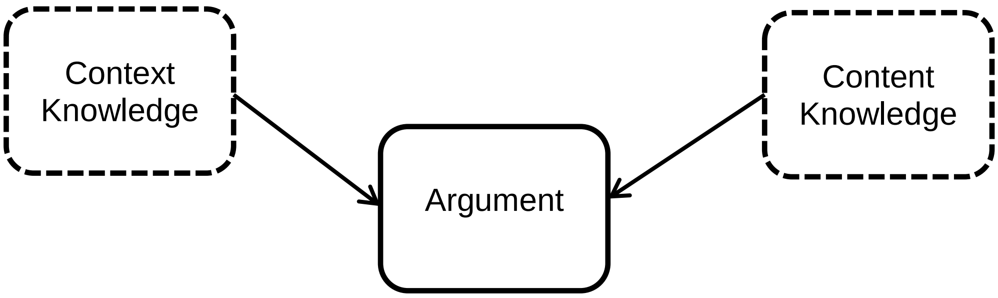


Figure 5.2: Initial impression of context and content influence on argument

Thus in planning, consideration was given to how each of these three aspects would serve the guiding principles of Knowledge Building (Scardamalia & Bereiter, 2006, 2010, 2013) that provided the philosophical underpinning of the planning of the interventions (sequenced units of teaching and learning). These principles are articulated in Section 2.4.2 in general terms. In the following tables, the way in which the planned intervention has been designed to meet the principles of Knowledge Building through argumentation (Table 5.3) and then through mathematics (Table 5.4) is presented.

Table 5.3: Alignment of Argumentation Practices with the Principles of Knowledge Building

Applicable knowledge building principle	How argumentation practices serves to address the principles of Knowledge Building
	Argumentation:
Real ideas and authentic problems	structure serves to situate the argument, and thus ideas and the evidence and reasoning behind those ideas, as real entities for debate and discussion.
Improvable ideas	necessitates the contribution of ideas; however, while ideas may be unformed or un-evidenced it is expected that ideas will be evaluated, with select ones being formulated as researchable questions. Evidence will then be sought to assess and work to improve, modify, or challenge those ideas, with eventually a claim, evidence and reasoning being put forth to address the question.
Idea diversity	benefits from diverse ideas in order to enable a multi-faceted approach to, and consideration of, problems
Rise above	practice works to obtain evidence and then organise, analyse and interpret the evidence to make an orderly presentation of this evidence and draw a conclusion (claim) from such.
Epistemic agency	practice positions the students to negotiate their own and other's ideas and to legitimise them with mathematical and contextual support. The students take the lead in the inquiry with the teacher providing some structure and guidance as needed. However the teacher's role is largely that of providing disequilibrium and challenging students to deeper ideas and understandings.
Community knowledge, collective responsibility	purposefully situates students so that their individual ideas are elicited and challenged, and establishes an environment of collective focus on developing a collective response that satisfies the community.
Democratizing knowledge	presents the opportunity for all students and adults to put forward ideas, provide evidence, and challenge evidence and reasoning, within a culture of developing 'best case' responses.
Symmetric knowledge advancement	does not necessitate symmetric knowledge advancement in and of itself, rather rhetoric can act against this principle. However, structuring the units so as to make each group accountable for contributing, and for challenging the contributions of others, serves to address this aspect: along with the adoption of epistemic argumentation as defined by Biro, Siegel and Lumer. (Biro & Siegel, 1992; Lumer, 2010; Siegel & Biro, 1997)

Pervasive knowledge building	practices are more aligned to the goal of pervasive knowledge building than learning pedagogies that seek to articulate carefully chunked pieces of knowledge to the students. However, the scope for the research here was across one subject only. Opportunities were taken by students to extend their knowledge out of specified mathematics lessons; however, it would be misleading to state that knowledge building was pervasive beyond the bounds of the research design.
Constructive uses of authoritative sources	is necessarily taught to the students by having students put forward claims and then drawing out through discussion the need for supports (evidence) to the argument claim.
Knowledge building discourse	necessitates its own genre-based terminology, structure, and discursive practices, which provide a basis for Knowledge Building practices. Epistemic argumentation is of itself a discursive practice which seeks to develop a collective explanation which best addresses the known evidence at the time.
Concurrent, embedded, and transformative assessment	enables insights into student's emerging understandings and allows those understandings to be challenged, assessed and deepened continuously. Further, the assessment originates with the audience, thus students receive feedback immediately, by both peers and adults, and in such a way as the feedback is specific, individual (to self or group), and may be queried or elucidated by the student presenting the argument.

Table 5.4: Alignment of Mathematics with the Principles of Knowledge Building

Applicable knowledge building principle	How mathematical knowledge serves to address the principles of Knowledge Building
	Mathematical Knowledge/Mathematics:
Real ideas and authentic problems	serves to provide a means for addressing real ideas and authentic problems and for providing the evidence required to make and support a claim, and the reasoning to link the two.
Improvable ideas	is treated as improvable, rather than a known body of facts: for example, representations are improvable through provision of certain, selected information, which the students must decide upon.
Idea diversity	benefits from diverse ideas in order to enable a multi-faceted approach to, and consideration of, problems
Rise above	is used to provide evidence and then provide the means to organise, analyse and interpret the evidence and to justify the decisions made while doing so. Mathematics is applied to a difficult problem and used to address those difficulties and create a solution.

Epistemic agency	positions the students to negotiate their own and other's ideas and to legitimise them with argumentation and contextual support. The students take the lead in selecting and applying mathematics with the teacher providing some structure and guidance as needed. However the teacher's role is largely that of instigating disequilibrium and challenging students to deeper ideas and understandings.
Community knowledge, collective responsibility	provides the conceptual basis and the language for the students to discuss and address the problem at hand, thus enabling a community to focus on a solution that is mathematically supported
Democratizing knowledge	provides the opportunity for all students and adults to put forward ideas, provide evidence, and challenge evidence and reasoning, within a culture of developing epistemically acceptable (ideal) responses.
Symmetric knowledge advancement	does not necessitate symmetric knowledge advancement in and of itself. However, structuring the units so as to make each group accountable for contributing mathematical evidence and reasoning provides opportunities for students to contribute their individual ideas, understandings and approaches.
Pervasive Knowledge building	enables the students to explore ideas outside of the traditional mathematics lesson, linking learning to contexts outside the classroom. There are inquiry topics that may better serve this principle if they are contextualised in more real-life problems.
Constructive uses of authoritative sources	is able to be obtained by the students, at request from the teacher, or of their own accord from textbooks, mathematical dictionaries, online mathematics sites and so forth. When the students experience difficulties they can draw on each other's successes and count on each other as authoritative sources at times. Unlike science, the children do not examine the authoritative source as they recognise that much of that they are checking is based upon agreed conventions rather than discoveries: for example, the naming of triangles is an agreed upon convention, not a discovery open to challenge or questioning.
Knowledge building discourse	necessitates its own discipline-specific terminology and conventions, which provide a basis for knowledge building practices. Epistemic argumentation is of itself a discursive practice which seeks to develop a collective, mathematically-based explanation drawing upon mathematically derived evidence.
Concurrent, embedded, and transformative assessment	understandings are able to be challenged, assessed and deepened continuously with the students receiving immediate feedback from the audience of both peers and adults, and in such a way as the feedback is specific, individual (to self or group), may be queried or elucidated by the student. The context also serves to provide feedback to the students as they work mathematically by setting parameters for reasonableness.

The Knowledge Building principles were also addressed by the context; however, the way in which this would occur was not able to be predicted or planned accurately in this instance as the students' prior context knowledge could not be predicted. Therefore, a table has been included to address the way in which the planned intervention met the principles of Knowledge Building through context (Table 5.5) but to some extent this has been completed retrospectively and, for the most part, addresses each context separately.

Table 5.5: Alignment of Context with the Principles of Knowledge Building

Applicable knowledge building principle	How context served to address/align with the principles of Knowledge Building	
	In the Barbie Unit, the context:	In the Pyramid Unit, the context:
Real ideas and authentic problems	linked authentically to the work and practices of applied mathematicians	linked authentically to the work and practices of pure mathematicians
Improvable ideas	required informal statistical inference and proportional reasoning to initiate representations that were continually enhanced to strengthen the evidence and reasoning	required the application of geometric knowledge and conventions to initiate representations that were continually enhanced to strengthen the evidence and reasoning
Idea diversity	required idea diversity primarily at outset while students negotiated the pathways to envisage a solution	necessitated idea diversity throughout to allow students to continually build, reflect and refine ideas
Rise above	resulted in data that was messy and required the organisation and interpretation of ambiguous and potentially erroneous scores that could only be interpreted through consideration of the context	led to initial ideas that were logical and organised and then transcended into messy ambiguity and challenge
Epistemic agency	was introduced with an ambiguous, ill-structured question which necessitated some management by the teacher to assist student to come to a consensus on methods	was less ambiguous, enabling more control to be taken by students within groups until the completion with different conjectures from each group
Community knowledge, collective responsibility	required a whole class solution developed from student groups and individual work to contribute to an overall class response – students took individual responsibility to contribute to whole class data for a whole class solution	enabled students to contribute to group knowledge with an overall whole-class sharing of ideas along the way and at the end – students built on each other's ideas but knowledge was collectively negotiated within groups
Democratizing knowledge	ensured each student had a unique piece of information to contribute – additional accountability was deemed necessary for the first unit for both summative assessment purposes and to ensure engagement	enabled less individual accountability and more group accountability – each group was accountable for contributing to the communities' understandings as a whole NB: some students (particularly two

	NB: some students (particularly two with Asperger's syndrome) had difficulty with the extent of the group work and were supported heavily by the teacher	with Asperger's syndrome) had difficulty with the extent of the group work and were supported heavily by the teacher in one instance and by their group in another
Symmetric knowledge advancement	was such that, while many members of the community suspected that barbie might not be in proportion, at least in some measures, no-one actually knew (even the teacher did not know which proportions would be out or to what extent)	was such that no-one in the community (including the teacher) knew the answer to this question at the outset – the teacher's geometric skills were merely further developed enabled students to work to contribute findings and developments to each other as they progressed, as well as methods
Pervasive Knowledge building	engaged students in activities by assigning out-of-school data collection tasks – not specifically knowledge building	ensured knowledge building was predominantly a focus of this unit and many students engaged in additional out-of hours contributory activities of their own volition – making multiple pyramids each night to share in the morning
Constructive uses of authoritative sources	did not require authoritative sources although some students elected to research versions of Barbie over the decades	leant support to research via books and internet to ensure all known Egyptian pyramids were regular
Knowledge building discourse	did not require specific contextualised discourse beyond those terms that would be considered mathematical: that is, proportion, length and so forth	was mathematical and therefore any knowledge building discourse was located within mathematics
Concurrent, embedded, and transformative assessment	enabled assessment that was formative and largely provided by one of two means: feedback from students and the teacher in the form of questions and challenges during explanation and persuasion phases and group negotiations; and feedback from the task itself	

5.6 Data Sources

Data collection occurred using several methods in order to obtain rich multi-sourced data. Data sources included:

Field notes: The purpose of maintaining notes was to keep an ongoing record of research observations, impressions, moments of insight, and developing questions. The nature of the research described was interventionist and as such the role of the researcher in this study is necessarily that of a participant observer (Flick, 2009, p. 226).

Videotaped lessons: Each of the class lessons was videotaped utilising the assistance of one or more of: a pre-service teacher assigned to observe the class, a researcher from the university who was also undertaking research as part of a larger project at the school, or a static camera set up in the classroom. The primary purpose of using video was to enable repeated viewings of lessons for the purpose of analysis. Video, as distinct from voice recording, assisted with identification of individual participants for cross-coding and allowed for nuances, such as facial expression and verbal and non–verbal cues to be detected.

Collection of children's work samples: Students, particularly young students, may find they have difficulty in articulating their understandings; however, their written work, including rough jottings, journals and sketches can further illustrate their developing understandings. Students' work may be open to misinterpretation if not coupled with an interview (Corbin & Strauss, 2008, p. 30).

Interviews with students: The need to interview students was based upon their articulated understandings, completed work samples and classroom discourse with the purpose being to follow through potential moments of insight, or to probe further into students' developing knowledge and understanding of evidence and argumentation. These interviews followed episodic and semantic forms of interview techniques. "Episodic knowledge is organised closer to experiences and linked to concrete situations and circumstances" (Flick, 2009, p. 185). Thus, episodic interviews were frequent, informal, and typically took place as the researcher casually approached and talked with students while they were undertaking tasks. As such, they were not typically structured but rather exploratory. These 'interviews' were captured on video tape and transcribed as part of the class lesson. On a small number of occasions, more detail was obtained through specific, semantic based questions (Flick, 2009). This second method enabled deeper exploration of learning activities and events while providing students the opportunity to make explicit links between concepts and identify relationships as they perceived them. These more focussed interviews were video recorded in small groups, rather than as individuals, to assist the students to build on each other's explanations and concepts.

5.7 **Data Analysis**

Grounded Theory (Corbin & Strauss, 2008) was used for the purpose of analysing, conceptualising and theorising data. Grounded Theory is a specific methodology that can be used for the purpose of building theoretical constructs from data (p.1), thus being well-aligned to the goals of Design-Based research which were explicated earlier (Section 5.3). The process used will be described below in a relatively linear fashion. While this reflects reasonably well the components of the pathway taken, there was a significant amount of reflection, adjustment and reconceptualising necessary to develop concepts more fully. This process enabled the establishment of various tools for classroom learning as well as furthering theoretical insights.

5.7.1 Procedure for analysis

Data collection was undertaken during the implementation of the teaching units. As the teaching was implemented, field notes were made and these formed the basis of a reflective data log to record initial impressions. These impressions included, for example: pivotal moments in the development of teacher or student understanding; pedagogical decisions made and the reasoning for such; struggles experienced by both teacher and students in moving forward; and, planning decisions that resulted from reflections on the day’s activities. It should be noted that the researcher logs for the first unit of work were destroyed when a natural disaster resulted in severe damage to several school buildings. The logs were retrospectively reconstructed from video data, children’s work samples, and discussions with key participants.

Reflecting on these broader elements of classroom activities, and student responses as they occurred, enabled consideration of potential directions for progressing student learning. Each lesson was thus responsive to the previous lesson, enabling reflection and planning, and thus supporting the reflective and prospective processes, as well as the interventions characteristic of Design Research (Cobb et al., 2003). This reflective process also assisted with the ongoing design (and often co-design with students) of potential classroom tools, such as diagrams and models, and practices that facilitated students’ progress and assisted challenges faced.

These initial notes and reflections served to enable the identification of early concepts (Corbin & Strauss, 2008) as these were fundamental to planning the approach of the next lesson. At this stage, concepts were general and memos and diagrams rough and impressionistic. A more detailed level of analysis did not occur until the entire unit had been taught as the time between lessons did not allow for this to occur in practice.

At the conclusion of each of the teaching and learning units, all classroom video recording was transcribed in full by the researcher in order to develop a cohesive overview of the unit and to engage fully with the context (as recommended by Corbin and Strauss, 2008). This was particularly important in this instance as the camera had often picked up student comments that the teacher had not, particularly when the camera was focussed on a group working without the presence of the teacher.

The transcripts for each unit were read in their entirety before coding commenced in order to develop an overall picture of the unit and embed the analysis in the holistic context. After this, each of the transcripts from the Barbie unit were subject to open coding. Sections that did not bear direct relevance, such as administrative aspects, classroom interruptions, and off-topic remarks, were disregarded; for example, students asking which book to record their ideas in. The remaining sections were interpreted, comment by comment, and coded using conceptual names (codes) sourced in one of three ways (Corbin & Strauss, 2008):

1. Theory derived concept names: those that were previously identified through the literature as being likely to be of importance; for example, ‘evidence’, ‘claim’, ‘question’

2. Analyst derived concept names: those that the analyst assigned as being representative of content; for example, ‘engaging students – driver’, ‘engaging students – provocative’

3. Participant derived concept names (in-vivo codes): those that the participants used themselves as being representative of content; for example, ‘evidence for the question’, ‘evidence for the conclusion’

The purpose of coding comment by comment, and using a mixture of code-name derivations, was that it enabled the analyst to "[put] aside preconceived ideas about what [they] expect to find in the research and [let] the data and interpretation of it guide the analysis" (Corbin & Strauss, 2008, p.160). Appendix C demonstrates a section of the coding names that were developed and utilised in this analysis, from categorical (higher, more abstract) level to lower level.

The use of memos and diagrams linked to these codes enabled reflection and eventual theorising about a structure of concepts from higher to lower level; that is, this reflection suggested the overarching abstract categories, as well as the underlying and more concrete ideas that went towards making up or describing these categories. This was then able to be used to identify general patterns and to form a working model established on a theoretical base, as theory creation was a goal of the Design Research implemented (Cobb et al., 2003).

Open-coding continued with the Pyramid unit until such time as saturation was deemed achieved. The transcripts for both units were then subjected to re-coding using these codes and code categories, in order to map themes and relationships (Clarke, 2005, p.83). This included the Pyramid unit transcripts not previously coded. The purpose of the initial open coding prior to content analysis was to assist in the identification of unanticipated insights (Corbin & Strauss, 2008, p. 160).

While a vague conception of content (mathematical) knowledge and context knowledge had been envisaged as supporting the production of argument (Figure 5.2), the interactions and components of each domain had not been considered or speculated on. This was in order to purposely remain open to findings. The use of coding enabled sub-aspects of each of the knowledge domains (mathematical, contextual and argumentation) to be identified, along with some insight into their roles. The roles and inter-relationships were largely establishing using cross-cutting techniques (Corbin & Strauss, 2008); that is, identifying the relationship of one concept to another. The greyed area of Appendix C serves to identify a small section of cross-cutting codes. One such insight was the extent to which each domain served to support other domains. The support of the context in the Barbie unit for developing the students' ability to visualise proportion was, for example, one way in which Context Knowledge clearly supported Mathematical Knowledge. Thus, the analysis resulting from the open coding led primarily to the development of the Triumvirate Model addressed in the conclusion.

5.7.2 Mathematical argumentation assessment framework

The research undertaken here encompassed a field which had received little prior attention. While argumentation practices in inquiry have been extensively studied in science education, the same cannot be said of mathematics. Multiple frameworks for the assessment of argument structure and quality have been proposed (Sampson & Clarke, 2006; Toulmin et al., 1984; van Eemeren & Grootendorst, 1992); however, none of these for the purposes of mathematics pedagogy. While any of these *may* have been suitable, it could not be assumed that what is epistemologically acceptable in science would be equally acceptable in mathematics. Thus, there was perceived a need to develop a framework for assessment of mathematical argument. This was developed retrospectively through analysis of student arguments.

Throughout the course of this study, the students collectively produced approximately 300 ‘arguments’ either as written artefacts or as spoken arguments captured on videotape. An argument was considered a communication that occurred as a response to the students being requested to provide such; that is, what the students themselves presented as an argument or a conclusion. Several of the argument transcripts and written argument artefacts were initially deconstructed using Toulmin’s argument structure. This however, proved impractical given the simple, though often disordered nature of young students’ communications. In particular, it was often difficult to differentiate grounds, warrants and backing from each other in practice; an issue that has been observed by other researchers (Erduran, Simon, & Osborne, 2004; Kelly et al., 1998). Given the limited complexity of the students’ work; the Claim-Evidence-Reasoning (CER) model (McNeill & Martin, 2011; Zembal-Saul et al., 2013) was felt to be more practical, and ultimately each argument was deconstructed into the CER categories and tabled, so that the component parts of the argument could be coded, without losing the argument as a whole. This was necessary to identify the extent of coordination between Claim, Evidence and Reasoning. In addition an ‘other’ category was used for any entries that did not serve the purpose of acting as Claim, Evidence or Reasoning.

The samples were open coded under the stems of Claim, Evidence, Reasoning and Other. Once a saturation point was reached at which no new codes had been used for some time (approximately 120 samples), open-coding was discontinued. Substantive categories were devised based upon the open coding. Other content areas were identified on the basis of their relationship to the categorical or higher-level concept. A tree diagram is included at Figure 5.3 to provide an illustrative example. In this instance, codes were identified for errors and omissions, which were collectively categorised as sub-concepts of accuracy, which in turn were linked to the students' representations of evidence. Finally as fourteen substantive categories were observed, these categories were examined and finally reduced to two over-arching categories: 'Argument Structure' and 'Epistemic Reference'.

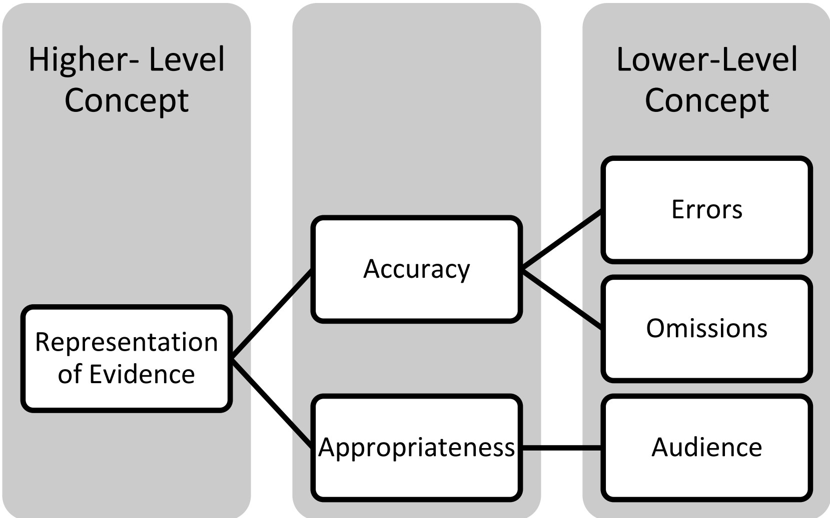


Figure 5.3: Example of coding levels for representation of evidence

As a result of this coding, criteria were developed which enabled students' arguments to be deconstructed according to either of the over-arching categories. The remaining sample arguments were then assessed against these criteria, with adjustments to descriptors made to encompass all of the variants and to ensure clarity in the wording. The final criteria are included in the following tables (Table 5.6: Assessment of Argument Structure and Table 5.7: Assessment of Epistemic Criteria).

While the design of the data analysis has been presented as a linear progression, it did not progress solely in that manner. As new features were identified that were of interest, it became necessary to return to previously collected data to attempt to make sense of it.

Table 5.6: Assessment of Argument Structure

		1	2	3	4	5
Research Question	Question the claim responds to	Question cannot be identified	Question can be inferred	Question is implied in text	Question is stated in general terms (may be contextual)	Question is clearly and specifically stated
Research Question – Context*	Question co-ordinates with the wider research context	Question and/or wider research context cannot be identified	Question does not co-ordinate with the wider context	Question co-ordinates with the wider context although these are not clearly delineated	Recognition that the question informs the wider research context	Question informs the wider research context, with explicit, appropriate explanation as to how
Claim	Claim states a clear, foregrounded position	Claim or claim position cannot be identified	Claim and claim position can be identified or inferred from text or evidence	Claim provided is • stated in general terms AND • lacks foregrounding May reference the context not the question	Claim is provided but is either • stated in general terms OR • lacks foregrounding May reference the context not the question	Claim: • is explicitly stated (including position) • is foregrounded **and** • references the question
Evidence (Grounds)	Evidence provided is relevant to the research question (	No evidence is provided or is irrelevant to the research question	Evidence is provided which is • tangential to the research question	Evidence is provided which is relevant to the research question but is • insufficient	Evidence is provided which is relevant to the research question and contains • sufficient detail with any omissions being immaterial • may contain unnecessary additional information	Evidence reflects audience, is directly relevant to the research question and contains • sufficient detail • without unnecessary additional information
Reasoning	Links from evidence to claim co-ordinate logically	Claim or evidence is absent and cannot be inferred	Claim and evidence do not co-ordinate	Reasoning requires inferring, or is non-specific, but claim and evidence co-ordinate	Reasoning is provided and claim and evidence coordinate logically but lack consideration of all evidence	Reasoning is provided, • co-ordinates logically **and** • considers all evidence
Claim – Context*	Implications for wider context are acknowledged	Claim or wider cannot be identified	Claim and/or wider context are absent but can be inferred	Claim and wider context are identified but implications are absent or lack relevance	Claim and wider context are identified Claim has implications for wider context though may be left to be determined	Claim implications on wider context is explicitly provided
Qualification	Identifies any limitations	No qualification attempted or is erroneous	Qualification needs to be inferred	Qualification is implied or needs to be inferred but the detail is sufficient to do so	Qualification is provided but may lack detail or be general in terms	Qualification is explicitly provided with details as to when it might apply

***Applies only if the research question is derived from a wider context**

Table 5.7: Assessment of Epistemic Criteria

		1	2	3	4	5
Epistemic References	Evidence collected / generated responds to the question being asked (Raw or unorganised)	No evidence is provided, or evidence is limited or incomprehensible	Evidence is irrelevant or only superficially relevant to the question addressed	Evidence responds to the question being asked but contains significant error or is insufficient	Evidence responds to the question being asked, is sufficient but with error	Evidence responds to the question being asked, is sufficient and contains no or minimal error
	Evidence is provided from data (as distinct from opinion, conjecture)*	No evidence is provided, or evidence is limited or incomprehensible	Evidence provided is opinion, conjecture or employs fallacy	Evidence relies on factually questionable material and may also contain opinion, conjecture or employ fallacy	Evidence derives from both a factual and objective viewpoint and relevant / reasonable conjecture	Evidence is provided from a factual and objective viewpoint
	Evidence appropriately gathered given community standards	Method of obtaining evidence is not provided	Method of obtaining evidence is provided without enough detail to determine appropriateness	Method of obtaining evidence is provided but contains significant flaws	Method of obtaining evidence is provided and is acceptable with limited flaws so as to not materially alter the evidence	Method of obtaining evidence is provided and is appropriate
	Evidence appropriately represented/ presented given community standards	No evidence is provided, or evidence is limited or incomprehensible	Evidence provided with no/limited attempt at organisation	Evidence representation is able to be understood; however contains errors that cause the claim to be affected, enables misrepresentation or is highly inefficient	Evidence representation is accurate and appropriate given audience and purpose with errors minimal and inconsequential, though may not be recognized as a standard mathematical representation	Evidence representation is accurate and appropriate given audience and purpose and is without error or omission
	Evidence appropriately interpreted given community standards	No evidence is provided, or evidence is limited or incomprehensible	Evidence provided with no /limited attempt at interpretation	Evidence interpretation demonstrates material flaws eg accuracy, clarity, method or efficiency	Evidence interpretation is not incorrect but does not meet community expectations eg clarity, method or efficiency	Evidence interpretation meets community expectations eg accuracy, clarity, method or efficiency
	Anomalous or contradictory evidence is accounted for* (if present)	No evidence is provided, or evidence is limited or incomprehensible	Evidence is provided without the inclusion of anomalous or contradictory evidence	Anomalous or contradictory evidence is included but not addressed	Anomalous or contradictory evidence is included and addressed through un-evidenced conjecture or supposition	Anomalous or contradictory evidence is provided and accounted for/addressed / explained factually or in terms of limitations
	Reasoning (evidence to claim) applied is appropriate in terms of community standards	Reasoning is absent and cannot be interpreted	Reasoning lacks mathematical acceptability	Reasoning is inferred or provided and indicates developing of epistemic understanding	Reasoning is provided which is epistemically acceptable	Reasoning is provided which is epistemically ideal

*Epistemic in nature as these requirements may not exist in all spheres of argumentation – for example, in a criminal trial, the role of the defence may include offering alternate conjectures for evidence that are not factually supported but may serve to invoke doubt by providing alternate interpretations of existing facts.

6 Results – Intervention 1

6.1 Chapter Overview

In the following chapters, the two units that are a focus of this study will be discussed in a semi-sequential manner. This is done to maintain the overall continuity of the students' experiences and thus enable the reader to follow the learning sequences and students' progress logically. Each unit in the study was planned specifically to have a role in developing an aspect of students' knowledge of argumentation structures and processes. This first unit, in which students address the question, "Is Barbie a human?", was specifically aimed at deepening the students' focus on the role and need for evidence in argumentation. The mathematical and statistical content was selected to meet state curriculum requirements, school year level plans, and/or student interest. A more extensive unit overview was provided in Chapter 5 to illustrate the focus of each unit in terms of the context, mathematical content, argumentation structures, argumentation processes, and teaching and learning sequence.

Two significant outcomes that emerged from the data collection and analysis are reported within this chapter. The first is the students' development and application of their knowledge of argument structure and the second is the way in which students applied the process of argumentation within the classroom. Overlaying both of these is the identification and acknowledgement of the students' emerging epistemic awareness: what it is that constitutes acceptable 'ways of knowing' within the field of mathematics. While it is acknowledged that 'epistemology' is a complex meta-construct, this study adopts the term in a more simplistic sense. Throughout the results and discussion, the term 'epistemology' will be used to refer to what is valued as acceptable knowledge and procedure within the field of mathematics, while 'epistemic reasoning' will be used to refer to the ways in which the students apply that knowledge to their learning. In all instances it should be noted that acceptable epistemic reasoning is relative to the development of the learners and the learning community. The aim was to develop ways of 'knowing' and 'doing' that are appropriate for a community of young student mathematicians: ways which increasingly approximate the mature discipline of mathematics.

6.2 **Argumentation Framework**

Argument is essentially both a product and a process: in order to be able to engage effectively in the practice of argumentation, students need to develop familiarity with both of these aspects. Argument as a product has a defined generic structure, while argument as a process has a defined purpose and generic norms. In both cases, the generic criteria are in part determined epistemically. What is acceptable or even desirable in some disciplines, such as the use of emotive devices in political rhetoric or advertising, would not be considered acceptable in others, such as science and mathematics. The former example values appeal to the people and a high degree of empathetic connection to the audience to be successful. These values would not serve the sciences well; where validity and reliability of evidence, logical connections, and a willingness to be challenged are highly regarded. Thus, students need to learn to focus on developing argument structure and presentation while also learning about what counts as acceptable evidence in the field of mathematics; or more specifically, in a community of student learners of mathematics.

The research undertaken here encompassed a field which had received little prior attention. While argumentation practices in inquiry have been extensively studied in science education, the same cannot be said of mathematics. Multiple frameworks for the assessment of argument structure and quality have been proposed (Sampson & Clarke, 2006; Toulmin et al., 1984; van Eemeren & Grootendorst, 1992); however, none of these for the purposes of mathematics pedagogy. While any of these *may* have been suitable, it cannot be assumed that what is epistemologically acceptable in science is equally acceptable in mathematics. Thus there was a need to develop a framework for assessment of mathematical argument. This was developed retrospectively through analysis of student arguments. A detailed description of this process was addressed in Section 5.7.2. Essentially, student arguments were deconstructed into the Claim-Evidence-Reasoning (CER) framework (McNeill & Martin, 2011; Zembal-Saul et al., 2013) and analysed using open-coding. Fourteen substantive elements were observed, examined and finally reduced to two over-arching categories: Argument Structure and Epistemic Reference. These are overviewed in Table 6.1 and provided in full in Section 5.7.2. These criteria have been used as a framework for assessing the students work throughout the research and identifying potential change.

Table 6.1: Criteria for assessing quality student arguments in mathematics

	Indicator	Descriptor
Argument Structure		
1	Research Question	The research question is clearly and specifically stated
2	Research Question - Context*	The research question informs the wider research context
3	Claim	The claim is explicit, foregrounded and references the question
4	Evidence (Grounds)	Evidence reflects audience, is relevant to the research question, contains sufficient (but not extraneous) detail
5	Reasoning	Reasoning co-ordinates logically and considers all evidence
6	Claim – Context*	Claim implications for wider context are explicit
7	Qualification	Qualifier is provided with details as to when it is applicable
Epistemic Reference		Within Community Standards:
8	Evidence Collection	Evidence collected / generated responds to the question being asked
9	Foundation for the Evidence	Evidence provided is data-based (as distinct from fallacy, conjecture, opinion)
10	Evidence Gathering	Methodology for obtaining evidence is provided and is appropriate
11	Evidence Organisation /Representation	Representation / organisation of data is accurate and appropriate for the audience and purpose
12	Evidence Interpretation / Analysis	Interpretation / Analysis of evidence meets community expectations: accuracy, clarity, method, efficiency
13	Evidence Anomalies or Contradictions	Any anomalous or contradictory evidence is provided and addressed factually or in terms of limitations
14	Reasoning	The justification for making a claim, based on the evidence, is suitable given the community of mathematical learners

*Applies only if the research question is derived from a wider context

The initial teaching-learning sequence commenced with a focus on the essential nature of evidence. In particular, the aim was to assist students to see the potential for evidence to address a problem, to envisage what evidence might be of use, and to develop a plan to obtain that evidence. Along with the need for the evidence was the need to consider epistemic acceptability and quality. These elements of the first unit addressed argument as a product and began to informally develop consideration of elements of Argument Structure. Issues central to the presentation of argument, that is, argument as a process, have been provided in Section 6.5, prior to details of the assessment of product and process in Section 6.6. It should be noted that separating argument process from product is arbitrary, as the process of presenting an argument necessitates argument structure to be observed; however, it is when the argument is presented to an audience (in written or oral format) that the audience has the opportunity to challenge the claim through the evidentiary basis and reasoning provided and this establishes a different set of conditions than when an audience does not exist (Berland & Forte, 2010; Berland & Reiser, 2009). Therefore the argument product and process are established as two categories for reporting of results. The separation and reporting of these categories will apply to each of the results chapters to facilitate the horizontal comparison of development across the teaching-learning units as well as vertical development within a unit.

6.3 **Argument as a Product**

At the most simplistic level, argument structure is usually determined as comprising claim and grounds. In order to justify making the link from grounds to claim, warrants and backing are added (Toulmin, 1958; Toulmin et al., 1984). However, this model can be considered overly complex when deconstructing students' arguments as it can be particularly difficult to distinguish grounds from warrants from backing at times (Erduran et al., 2004; Kelly et al., 1998). Thus the Claim-Evidence-Reasoning (CER) model (McNeill & Martin, 2011; Zembal-Saul et al., 2013) was adopted both as a model for the students to follow and for the purpose of deconstructing arguments. The following sections address the students' experiences with developing an evidentiary focus. This addresses students' developing appreciation of the need for evidence and then students' emerging understandings as they develop through encounters with epistemic criteria: gathering evidence, interpretation and analysis, and representation of evidence.

6.3.1 Introduction to the evidence model

Prior to the commencement of these planned research units, the students had been briefly introduced to the Evidence Model (Figure 6.1). While appearing deceptively simple, previous research by the author had shown it to be highly effective in assisting a different cohort of same-age students to plan, envisage and answer inquiry-based problems (Fielding-Wells, 2010). Brief introduction of this model had previously been made to the students in this study, although the teacher felt that the students had merely adopted the terminology without a deeper understanding of the model itself.

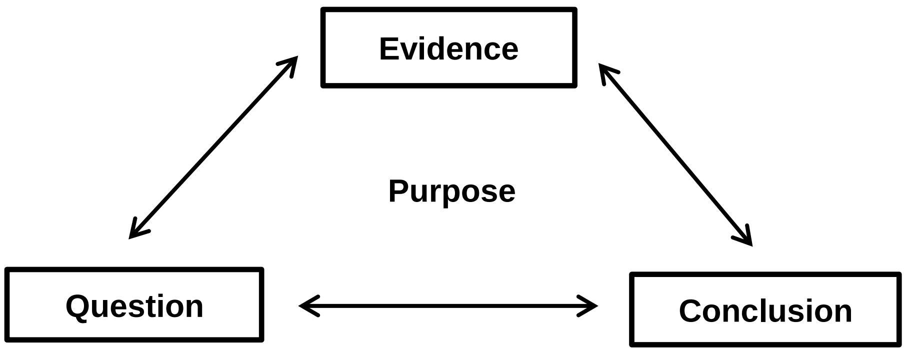


Figure 6.1: Evidence Model (Fielding-Wells, 2010)

Evidence plays a principal role in argumentation; once a question has been determined, there is a need to envisage the evidence that will be required to address the question and to formulate a plan to obtain relevant evidence. Once gathered, the evidence must be organised and analysed to enable a justifiable and co-ordinated claim to be made. Thus it is essential that students appreciate both the important role that the evidence plays, as well as seeing a need for it to be acceptable to the community of learners engaged in the inquiry. In this first inquiry, the focus was initially on ensuring students could see a requirement for evidence, and secondly, to help students to determine that the evidence must be acceptable in terms of the mathematical discipline, in relation to this specific community of learners.

6.3.2 Focusing on evidence

To generate the inquiry, the students were introduced to an initial, intentionally broad, question; ‘Is Barbie a Monster?’. In this section, we see the students address this question and work past their initial impulsive responses to focus on the evidence that might be needed to provide a solution. The teacher commenced the lesson by asking students whether Barbie was human. Based on previous research in science inquiry, it was anticipated that students would provide initial responses that were unfounded, and lacking in evidence (Muller Mirza et al., 2009). To illustrate, McNeill's CER framework (McNeill & Martin, 2011; Zembal-Saul et al., 2013) has been used below to deconstruct the first of students' simple arguments and to examine the extent to which their claims provided or lacked support.

1.	T:	Here I have a Barbie doll and I am wondering, 'Is Barbie a human?
2.	Delmar:	No way!
3.	T:	Why not?
4.	Delmar:	She is made of plastic.
5.	T:	That tells us she isn't real?
6.	Delmar:	And when you look at her she doesn't talk.

In the excerpt above, Delmar's initial response (claim) was unsupported (2); however, he did not hesitate to provide his evidence when prompted (3-4) and continued to add to it when challenged (6), indicating that while his responses were virtually immediate, they were considered and based on observable characteristics and existing knowledge. This response, rather than showing a lack of evidence, drew on the evidence which Delmar had available to him; that of observation. A response, 'No, she is made of plastic', cannot be stated definitively as being impulsive or lacking in evidence. The student has made a claim (Barbie is not human) and evidenced this (she is plastic), while implying reasoning (humans are flesh and bone). This argument has been deconstructed according to McNeill's framework in Figure 6.2 to enable discussion around various components.

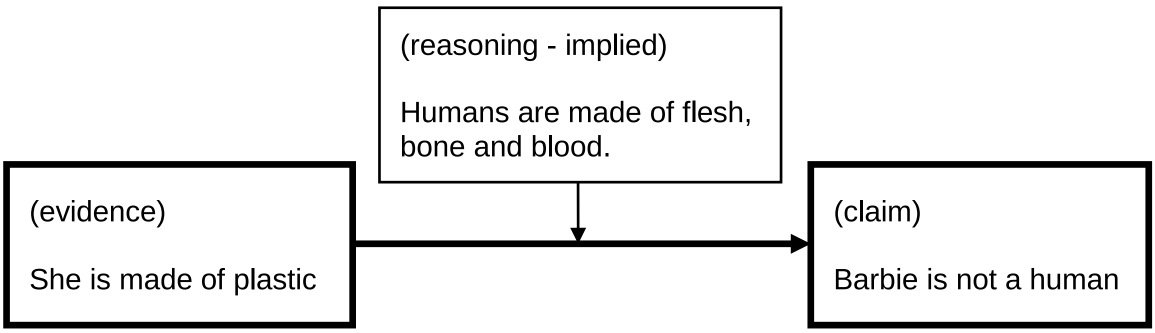


Figure 6.2: Deconstruction of 'impulsive argument'

To assess the validity of Delmar's argument, the structure and the acceptability of the argument need examination. While the argument is logically structured, with coordination between Claim-Evidence-Reasoning, the reasoning lacks the mathematical basis desired in a mathematics class. Consider though that there was nothing in the question to act as a pointer to students that a mathematical response was required. Without such a mathematical focus, the potential existed for the mathematics to be lost to the context (Allmond & Makar, 2010). Thus the question required modification and refining to incorporate a mathematical pointer: something that would suggest to the student a need for mathematics. The teacher worked with the students to refine the question with the goal of facilitating connections with mathematics and to envisage what mathematics might be useful in terms of a practical application:

7.	T:	Do we need to narrow down this question a bit?
8.	Dominica:	I think we can all see that she doesn't walk but we are talking about the way she looks.
9.	Shana:	You mean if she was a human would she be like a human?
10.	T:	In what way?
11.	Lee:	Like her face is the same as that [pointing to facial proportions diagram from the morning art lesson].
12.	T:	So if she was real, would her face meet these proportions? [pointing to same diagram]
13.	Connor:	Does Barbie have the same proportions as a human?

Shana's observation (9) more closely approximated the teacher's intended mathematical foci for the unit (proportional reasoning and statistical inference) enabling the teacher to privilege the comment and lead the students to rephrase the question; "*Does Barbie have the same proportions as a human?*" (13). This new question incorporated a clear mathematical pointer to proportion. Discussion ensued around Connor's question and the class decided that this was a more precise question and better reflected the intent of the wider context. The students turned their attention to the newly refined question; once again, responding quickly:

14.	Dominica:	She could kind of be looking like her eyes are not halfway from her crown to her chin.
15.	T:	Are they?
16.	Sts:	[Chorus of yes/no]

17. T: Has anyone checked?

18. Dominica: I looked and I think they are up too much.

19. T: A *bit* high? Mmmm..they *look* like they are….. Anyone have a suggestion?

20. Shana: Her eyes are too big.

21. Teacher: Are they? How do you know that?

While the evidence is again of an observational nature, the reasoning (see Figure 6.3) made a corresponding shift towards a mathematical foundation. While the reasoning is implied rather than articulated by the students, the deconstruction illustrates that there is now potential for the mathematical underpinnings to be developed: as the reasoning reflects increased proportional thinking, a greater focus can be put on obtaining evidence through the application of mathematics. From the interaction noted here, it would appear that the presence of a pointer in the question (proportion) may be significant in assisting students to identify and maintain a mathematical direction.

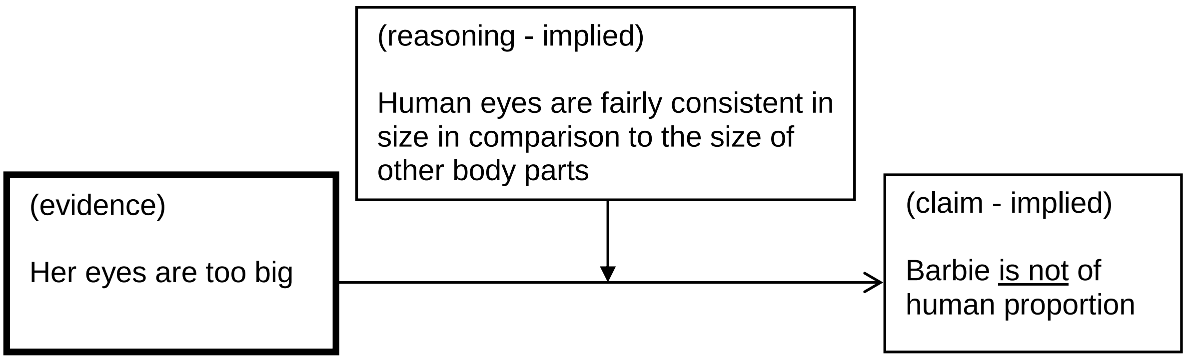


Figure 6.3: Shana's argument deconstructed

Class discussion continued around proportion and the teacher continually challenged the students to back their statements (lines 15, 17, 19, 21), waiting for the opportunity to lead students to envisage the need for evidence to support their ideas. However, it was when two students disagreed that a resolution appeared to become more important to the class and served to focus students more effectively than the teacher's comments.

22. Shana: Well, her neck is not normal, it is too long.

23. T: Why do you say that?

24. Shana: It looks too long.

25. Dominica: No. It looks about normal.

26.	Oliver:	We could test it.
27.	T:	How?
28.	Oliver:	Someone could like bring in Barbie dolls and we could get into our groups and we could look at it and we could estimate if she was a human height whether she would be normal.

This minor and amicable disagreement was to resurface repeatedly throughout the unit. While not engineered by the teacher, it supported previous findings in science education that cognitive disparity can serve as a driver to argumentation (Asterhan & Schwarz, 2007; S. Simon & Richardson, 2009), and this includes student-created disparity. By disagreeing, the students had now identified an issue that could not simply be resolved by observation or explanation and it drove the need for obtaining evidence (26-28). Oliver's suggestion to 'test' the length of Barbie's neck provided an opportune student resolved segue to a deeper focus on mathematical evidence.

So far in the unit, the students had deepened their engagement with the context, narrowed down their research focus under the teacher's guidance, and had seen the need to gather evidence. Once the students had made a shift towards more evidence-focussed thinking, the teacher determined to have students consider what evidence would be acceptable in a community of learners of mathematics and to try to envisage that evidence as it applied to their inquiry question.

6.3.3 Envisaging and planning to obtain evidence

When attempting to address a problem, it is necessary to envisage the evidence that is required to enable students to plan methods to gather or obtain such evidence (Krajcik et al., 1998). In the next section, the nature of evidence was addressed in terms of what would constitute evidence in mathematics (as distinct from other forms of inquiry such as historical inquiry, or from intentionally biasing forms or argument, such as advertising texts).

Students are likely to have been introduced to persuasive text writing early in their primary schooling, a genre which accustoms them to stating a case and defending it from a single viewpoint. Thus there was a strong expectation that children's conceptions of argument would stem from an expectation of supporting a claim with fact and/or opinion as distinct from deducing a claim from findings. The teacher aimed to elicit students' understandings about the role of evidence in the context of mathematical inquiry and to consider how students envisaged purpose of evidence co-ordinated with its mathematical argumentative purpose.

29.	T:	So I'll ask you, what would you need to do now?
30.	Sadie:	Find evidence?
31.	T:	Why do we want evidence?
32.	Gemma:	To support our conclusion **so we can answer the question properly.** And we know it's true. So **to prove.**
33.	Shana:	We need the evidence so that we can **back up our conclusion** so people can believe that what we are saying is right. …If we go straight from question to conclusion, we say that Barbie doesn't have human proportion and that's the end because you don't have any **details**. You don't know if it's her arm or anything that isn't in human proportions.

The viewpoints of the students suggested they perceived a distinction between advocative and inquiry approaches to argument (Toulmin et al., 1984). An advocative-argumentative approach focuses on providing support for an existing premise. These ideas align with the generic structure of persuasive texts often taught in schools. An inquiry-argumentative approach by contrast would foreground the evidence as the source from which the claim or conclusion is drawn and justified. Comments such as *"so we can answer the question properly*", "*to back up our conclusion*", and the consideration of evidence to provide the detail, are all indicative of an inquiry approach. What this does point toward is that the students may already be aware of both inquiry and advocative argumentative approaches, though may not clearly distinguish them in their minds. In this instance, Shana and Gemma appear to recognise the purpose of the evidence they were gathering.

As the students were developing a rudimentary understanding of the need for evidence in the inquiry, the teacher tasked them to develop group plans for the collection of evidence: evidence which would enable them to support their claim regarding the 'humaneness' of Barbie's proportions. The purpose of this activity was to determine the extent to which students could initially envisage a pathway that would support the evidence gathering. The groups then reformed as a class to share their ideas. The below example illustrates one group response.

34. Gemma: [reading aloud what her group had written]:
Number 1: measure human proportions.
Number 2: measure Barbie's proportions.
Number 3: Compare Barbie and human proportions and see if they're close: like measure head size and work out if the eyes are halfway and do same with Barbie and see if the proportions are close.

The students had little difficulty with planning to obtain data in general terms, and were consistently able to provide a plan that indicated they were cognisant of the need to provide some form of evidence in order to address the question. As the groups completed and then later presented their proposals for obtaining evidence, questioning from the teacher and other students identified areas of struggle. While some groups became fixated on smaller issues, such as what to measure or what type of doll should be used for comparisons, every group struggled with how the comparison would actually be carried out and what comparisons should be made. Attempts to move the inquiry forward were unsuccessful until the teacher attempted to break this position by having students imagine what a conclusion to the question might 'look' like, and to write an envisaged possible claim along with what they thought the supporting evidence might be. The students were provided with a verbal claim to act as a stem; "Barbie has human proportion because…" or "Barbie does not have human proportion because…". The secondary purposes of this activity was to begin to introduce students informally to the basic claim-evidence underpinning of argument structure, particularly as research suggests that students experience significant difficulty with the co-ordination of claim and evidence (Sampson & Clark, 2008).

The students' statements were deconstructed into claim and evidence, open-coded and reported as below. Unsurprisingly, all students except one (a student with expressive language difficulties) were able to express their claim in roughly the same language as that which the teacher had provided (Table 6.2) with one student even adding in an unanticipated qualifier, recognising that the claim was not definitive as it only addressed one ratio among many.

Table 6.2: Students ability to envisage a claim (n=26)

Claim	Number	%	Example
Qualified (Mentions test subject, population, relationship, and parameter)	1	3.8	Barbie has not got human proportions <u>for every body part.</u> [Gemma]
Relevant & Complete (Mentions test subject, population, relationship)	23	88.5	Barbie doesn't have human proportion [Konrad]
Relevant & Incomplete (Mentions relationship but assumes test subject or population)	1	3.8	Barbie des'nt have the same proportions [Seth]
Unclear	1	3.8	Barbie does not have the same body of a human body. [Darell]
Total[a]	26	100	

[a] The total number of students in this class was 27 - in any instance where the number of responses is less than 27 it is due to student absence for the activity unless otherwise specified

While the students were provided with a stem for their claim, their responses regarding envisaged evidence were unguided. These responses were coded and five distinct categories were identified. Often students provided more than a single source of evidence and when this was the case, answers were deconstructed and scored at the highest level of proportional reasoning to demonstrate the student was moving towards developing that level of understanding. This was also done to prevent data skewing as an effect of counting a student's multiple contributions from the same category more than once. These are shown in Table 6.3 below with illustrative samples for clarification. Evidence which illustrated *explicit proportional reasoning* describes those responses that gave clear focus on proportion (for example a ratio or fraction) even if informal and limited to one aspect, whereas *implicit proportional reasoning* refers to instances where the students' comments indicated they were thinking multiplicatively but without being explicit. In both instances, these responses are deemed to co-ordinate with the claims made. Those statements coded as *additive* indicate that the students were using additive conceptualisations of the differences between human and doll rather than multiplicative. The *methodological* category is inclusive of statements that cover the ‘how’ of the collection of evidence rather than indicating what the evidence would be. Finally, the *other* category incorporates mention of physiological differences between Barbie and human which cannot be measured proportionally in this context. The students’ responses enabled the teacher to identify whether they could envisage the evidence that might support a claim effectively and whether they were beginning to think multiplicatively. Overall, the students’ evidence suggested that many of them did not as yet conceptualise the mathematics that underpinned the nature of the evidence; that is, proportional reasoning.

Table 6.3: Evidence students envisaged would support their imagined claim (n=26)

Envisaged Evidence	Number	%	Example
Explicit Proportional Reasoning	10	38.5	It’s knee isn’t halfway down the leg [Connor] Halfway down from the nose to the chin is the middle of your mouth and barbie is the same [Cho]
Implicit Proportional Reasoning	6	23.1	Her arms are the same size of a human if we made her to size [Sadie]
Additive	2	7.7	Her feet are 3cm longer than the human proportion [Seth]
Methodological	5	19.2	We measured Barbie and then we measured a human then shrinked the human and put both proportions on a piece of paper and compared [Andrea]
Other	3	11.5	Her ear only has one bump [Konrad]
Total	26	100	

This activity did serve to further both group discussion and whole class discussion by bringing forward both the mathematical context of proportion, which was to later provide the mathematical content of the unit, and aspects of methodology. As students had not had feedback on the ‘correctness’ of their responses, the students did not appear constrained about sharing their answers and proceeded to challenge each other’s ideas. In particular, the discussions addressed:

- The ‘sort’ of evidence which served to address proportion as distinct from, for example, number of toes;
- Discussion on proportional reasoning rather than additive reasoning; and,
- The need for deepened mathematical responses, for example shifting away from a qualitative statement towards those with a more quantitative focus.

In addition, several students had considered methodology rather than evidence. This also came up in student conversation and the students gave consideration to what would be required to obtain the data. Imagining a response appeared to provide the students with the content for re-engaging in discussion; consequently, they proceeded to plan their approaches and to share these plans with the whole class for comment and feedback, including any difficulties they envisaged.

After class sharing of the plans, it became apparent to students that their collective understanding of proportion was limiting their ability to continue.

35.	T:	OK we said body and head proportions as well, didn’t we? So that was your investigation question that we were going to have a look at. What do we need to do next?
36.	Oliver:	Work out the proportions.
37.	Shana:	We don’t know how to.
38.	Gemma:	And you still need to teach us the proportion stuff.

In response, the teacher employed direct teaching methods to explain proportion using models of unifix cubes (connecting cubes) both with and without comparison to human features, fractional representation and use of the algorithm with interpretation of the decimal answers. This just-in-time teaching was intended to further the students’ understanding at the point at which they would have a need for the mathematics. While this was a more teacher-centred lesson, it was noteworthy that students were comfortable with questioning the teacher and each other. While the teacher had an appreciation for what needed to be addressed, it was largely the students’ responses and questions which determined how long was spent on each aspect of the lesson and the representations used. The teacher moved between the context and content, varying the degree of abstraction in response to the students. The students applied their newly acquired appreciation for proportion to refining their plans and moving forward with the collection of human data for comparison.

6.3.4 Envisaging the data collection

A long term aim to argumentation development is to have students consider not only the need for evidence, but the quality of that evidence. One significant contribution to evidence quality comes from the methodology employed in its collection. How data are collected, issues of validity and reliability, and whether the design or methodology is considered when evaluating the evidence, are all issues that require addressing when considering the strength of the evidence (Sampson & Clark, 2008).

The teacher had not originally planned to address data collection closely; however, there were two instances that she felt were opportune teaching moments to introduce important aspects of data gathering that had specific application to this unit: consistency in measurement and the issue of sample size. The first opportunity presented itself when a group of students found what they believed to be a common ratio between overall height and height-to-thigh measures. However, the teacher had observed the students were measuring each other's height, halving it and then checking to see if that was the location of the thigh. As the thigh is considered a fairly long section of upper leg, it was unsurprising that the midpoint always occurred somewhere within that section.

39.	T:	OK, I want everyone in the room to stand up. Close your eyes. I want you to put your hands on your thighs. So place your hands on your thighs so your fingers are touching your thighs. Keep your eyes closed. OK, freeze. Open your eyes and have a look around the classroom. OK, we have a range of people that are touching spots from ankle to hip bone; oh, we have a waist as well. So, somewhere between the ankles and the waist you will find your thighs. Is that specific enough? So for these things we might have to decide exactly what we are talking about.

This quick demonstration was intended to help the students see the need for consistency, preciseness and uniformity in measuring and data collecting and provided a favourable moment for the teacher to demonstrate the need for attention to be paid to a need for accuracy. This same discussion led the teacher to introduce the second issue which would become relevant in future statistical work, that of sample size. During planning discussions, many of the groups had been talking about comparing the Barbie doll to a single human and students needed to see that a sample size of one was insufficient to represent the population; however, this did not arise as a student-originated issue during planning. The teacher waited for a natural moment:

40.	T:	So who did you measure when you got that your thighs were halfway down
41.	Gemma & Luna:	Everyone in our group.
42.	T:	And it was the same on all of you?
43.	Group:	Yep.
44.	T:	So, on me, would my thigh be halfway down my body.
45.	Gemma & Connor:	Maybe
46.	T:	So do you think measuring three people in your group is enough?
47.	Connor:	No
48.	T:	I am just wondering. How many people would you need to measure?
49.	Gemma:	We might need to measure different people with different heights.
50.	T:	All right, so that is something you will need to work out: How many people you want to measure before you can say, ‘Yes, the thighs are halfway down the body’.

While the teacher’s purpose in this discussion had been to raise the issue of sample size, Gemma (49) focused on the mathematical intent behind the expectation of increased sample size, that of consideration of variability. Statisticians would not accept a sample size n=1 in such a context because of the possibility of the selected individual not being representative of the wider population: a less likely situation when a large sample size is adopted. One of the aims of adopting an argument-based pedagogy is to assist with the enculturation of students into the epistemic basis of the more mature discipline.

6.4 Epistemic Criteria for Evidence

The nature of scientific and mathematical epistemology dictates that a central role is taken by evidence. These disciplines suppose that claims will be made, after examining findings or data, in order to advance theories or solutions. Analysis of the students' work over the course of the two units enabled the identification of specific epistemic markers that could be used as criteria for the acceptability and quality of mathematical evidence or evidence-getting. For further detail of how these criteria were established, refer to Section 5.7.2. These criteria addressed the epistemic acceptability of the way in which students:

- Gathered evidence
- Organised and represented evidence
- Interpreted and analysed evidence

These three criteria are addressed in the subsections below, with excerpts of students' work, classroom conversations, and/or assessments by way of illustration.

6.4.1 Gathering evidence

Often in an inquiry activity students would be afforded the time to collect their own data and then reflect later on the process. However, in practical terms there was potential for problems in asking students to measure adults' proportions without careful direction. Thus, the teacher assigned students a series of adult human measurements to collect from their *own* parent (parents had previously been contacted with regard to this and no measurements were taken that involved any part of the trunk). Discussion also took place about marker points on the body, for example, where the crown of the head is located, where to measure the knee from and so forth. The students collected these data for homework and returned them to school within a week. Through class discussion, the students made the decision to each take responsibility for one ratio and to compile and work with all the collected data for that ratio; for example, height: head height. Once this was done, the issue of analysing and interpreting the data was able to be addressed. Because this stage of the inquiry was teacher-led, there was little in the way of epistemic development or conceptualisation to report.

6.4.2 Representing and organising the evidence

In authentic problems, there is often more information (evidence) gathered than is required to address the problem. This is because we typically don't know exactly what we will need to address a problem. When addressing real problems, it is often necessary to gather the information and then organise it and apply a filter as to what is useful. Often there are successes and failures that result in data being produced that are irrelevant or not required, at least at a particular time or in a particular context.

This section describes the repeated iterations students undertook to organise the raw data they collected and collated. The purpose of the levels of iteration was to observe changes that occurred in students' work as they shifted focus from Sense-Making to Explaining to Persuasion (Berland & Reiser, 2009). In the first iteration, students were explicitly asked to provide evidence to address the question, "What is the human proportion for the measurement you have?", for their own understanding or Sense-Making. This was done in their own mathematics notebooks so as to increase the sense of personal understanding. In the second iteration, students were asked to put a response on a poster that could be displayed in the classroom, that would make it clear to others what they thought the range of human proportions for their measurement was. Students were explicitly told that other students would need to understand the answer and evidence from their poster, but that these posters would be put on display and so there would be no opportunity to answer questions. That is, the posters must standalone in Explaining (Berland & Reiser, 2009) their claim and evidence. After some discussion around what was effective in explaining and making a response clear, students were invited to create a third response which they would use to address the class, explain their response and be challenged by the class (including the teacher). In this instance, students were explicitly told they would need to persuade others that their claim and evidence were 'correct'.

In each instance, the students' responses were collected and coded (this was done as part of the overall coding of arguments as described in Section 5.7.2). The responses provided by the students were categorised into like themes to provide an overall sense of students' thinking and progress. At the end of the chapter, a comparison between students' first and final poster responses is provided. This is based on the assessment of the students' arguments against the Argumentation Framework provided in Section 5.7.2.

First Iteration

This section commences with the students' initial attempt at determining a human proportion from their data. After receiving their individual raw data, the students were challenged to look at their data and answer the question, "What is the human proportion for the measurement you have?", and "Why do you think so?". A summary of student responses is provided below at Table 6.4. The students used a variety of methods to determine an 'answer' from their data. While some were unrealistic in their range and had clearly incorporated extreme outliers, the students for the most part demonstrated rational reasoning. The reasoning cited most frequently involved either the use of the extreme data points, or values close to those points, to create a range, or alternately was a single score based on the data mode.

Table 6.4: Students' initial responses to human proportion question

Answer	Focus	No.	Example
Single answer	mode	8	because 1.0 was the most common of the ratios [Delmar]
Range	bi-modal	1	7.0 – 7.7. Because they are the two most common proportions out of all the proportions I have recorded. [Salome]
Range	lowest and highest values	7	1.0 – 2.4. All I did was got the lowest and the highest ratio and put them in a range. [Seth]
Range	lowest and highest values - narrowed	4	1.6 – 1.8. It is the most populist part of the measurement. [Cho].
Range	lowest and highest values - widened	1	0.8 – 1.3. [no explanation but had selected one position above and below the full data range]. [Leticia]
No response			
Total (n)		23	

At the completion, students were requested, on a scale from 1 to 10, to rate the extent to which they felt their answers were 'correct' and then to justify their score (Table 6.5). The purpose was to challenge the students' early analysis and engage them in discussions around statistical approaches they could take to make sense of their data.

Surprisingly, only eight of the 23 students looked to their data interpretation to justify the "correctness" of their responses, and only half in total maintained a focus on data at all.

Table 6.5 provides an overview of students' focus with examples of comments according to category. It was concerning that students were not relying more heavily on the data and data analysis as a means of supporting their claim.

Table 6.5: Source of student confidence in their initial attempt to determine a human proportion from a sample

Focus	No.	Examples
Data Analysis and Interpretation	8	• It was just choosing a highest and lowest number some might of not been an average number. [Seth] • I am not egsactly shure that the way I did it was correct. The way I did it was by takeing the most popular number and there were 2 numbers that were the most popular so I did both of them. [Salome] • I think it could be right because my way of the most common ratios is right. [Delmar] • I think I may not be right because I am only seeing data from 13 people and there may be more people with different proportion. [Andrea]
Data Representation and Organisation	4	• 'maybe usded the wrong way to work it out by using tallyes' [Sadie] • I don't think tallys is enofh. [Kody] • I thing I am prey sure that I am right because I put it on a dot plot and that made it easiyer to see [Leticia]
Data Gathering	1	• I got it of real information [Konrad]
Method - Unspecified	2	• I am not sure if I have used the right method [Gemma] • Some of my answers might be right and wrong [Denise]
Own understanding	1	• I think about 50:50 because I can't figure it out [Lee]
Effort	3	• I think it is about 50:50 because I haven't done my worst or my best [Lee] • I think I can do better [Cho] • I don't think I did much working out [Kody]
Affect	1	• I think I have 10 [the highest confidence rating] because I care [Geneva]
No justification	7	• I just don't think it is right [Leanne] • I am really sure but not absolutely certain that my answer is right [Zachary] • no comment at all
Total students	27	

In an attempt to focus the students back on the necessity of evidence, students were asked to what extent they thought they would be able to convince others that they had the ‘correct’ answer; again through rating and explanation. Once again the results were unanticipated; there was even less focus on the data and data analysis (Table 6.6). Rather, students’ focus noticeably shifted to the audience and the ability of the audience to understand the information provided, or to whether the right amount of information had been provided. This was partially reassuring in that audience consideration is an important part of argumentation (Berland & Forte, 2010); how much information we give, how it is displayed, and the level of mathematical and statistical analysis presented would vary depending on the audience. Yet, while audience is important to all of these considerations, evidence must necessarily be based on valid, reliable data. As only one student explicitly referenced the role of the data (Leticia), there was clearly a need to undertake further work on the nature of evidence and this was incorporated into the design of the ensuing lesson.

Table 6.6: The extent to which students felt their claims would convince others

Focus	No.	Examples
Data Interpretation and Analysis	1	It can be 1.6 instead of 1.7 because it is the nearest [Hailey]
Data Representation	1	I made this on a dot pot I crierkled [circled] the custer and you easy see whats the poplerst [Leticia]
Understanding [Audience]	7	Because I don’t really know if they will understand it all [Connor] Because I don’t think people will get what I mean on my page [Laverna]
Accuracy	1	People will believe me because I correctly wrote down everything that is on this page onto the piece of paper [Geneva]
Amount of information provided	10	Becaues I haven’t put much information on my paper [Sadie] I think it is 10/10 because I have put heaps of information [Lee] I think I gave them enough information [Sally]
Egocentric viewpoint	1	I think the people who are reading it will think the same as me because I think the way I put it down on the paper was the way that I understood it so they would understand it too [Salome]
No justification	6	People would think it is an OK answer [Seth] They might think some of it is convincing and some isn’t [Denise] Or no comment at all.
Total students	27	

Second iteration

At the teacher's request, the students again addressed the question they had responsed to in Iteration 1, but this time the students were provided with a sheet of A4 paper (which remained unnamed and was numbered only for research use). This time, students were advised that the purpose was to provide the claim and evidence for the same ratio to their classmates with the explicit instruction that the classmates would not have the opportunity to ask questions; therefore, the information had to be sufficient for them to see the claim and evidence. These pages were known as 'A' posters to the students and have been labelled as such in any following discussion. The purpose of this poster was to shift the communication from internalised understanding (or tacit communication) to that which required explanation (rhetorical communication) (Berland & Reiser, 2009; van Eemeren et al., 1996), it was anticipated that students would need to rely on evidence more effectively in the knowledge that their explanation needed to be articulated effectively to others.

After these posters were completed, and in order to assist students to develop a more objective focus on the data, the teacher decided to have students view each other's work and consider what was useful and what was not in terms of being convincing. The students were asked to pass the posters around the class and to specifically make suggestions for improvement to make the work they were critiquing more convincing to themselves as audience. These posters were passed around several times, students made comments on the obverse, and the posters were given back to their authors. The comments provided the students with feedback as to how convinced the reader was that the range of 'normal' human proportions provided was accurate and supported by evidence.

After the opportunity to critique several responses, a class discussion ensued and the students established a set of guidelines to work from. Two lists were generated by the students; those limitations that prevented the work from being effective, and those aspects they saw in people's work that they thought were particularly effective. This list was recorded by the teacher on the white board. Thus, the ideas are the students' but the phrasing may have been slightly adjusted by the teacher for grammatical purposes. However, meaning has not been altered. These points were left on display in the classroom for students to refer to.

The student generated lists are provided in Tables 6.7 and 6.8. The first column is the aspect identified by the students and the remaining columns indicate the Epistemic Criteria that the guideline would stem from (added later after coding of arguments identified these categories). The critique made by the students focussed on three key criteria: Data Interpretation and Analysis; Representations and Organisation of Data, and Communication of Data.

These responses provided interesting insight into the students' perceptions of argument. When students were asked what they found that gave them personal confidence in *understanding* the answer, 12 of the 27 students ('Data Analysis and Interpretation' and 'Data Representation and Organisation' in Table 6.5) provided an answer that was associated with one of the identified Epistemic Criteria. Yet when the focus shifted to convincing others, there was a decided shift away from Epistemic Criteria towards issues of audience (Table 6.6). This would tend to suggest that students see something other than epistemologically acceptable evidence as being important when convincing others. Ironically, when the students became the 'audience' that needed convincing, and critiqued each other's work, they argued that what they found convincing was actually aspects of the Epistemic Criteria (Tables 6.7 and 6.8). The teacher hoped that this would assist students to recognise the importance of the criteria; however, even if it did not, the checklist was a useful tool to refer students to.

During the class discussion on what was effective (or not) in convincing students of the accuracy of claims and evidence, another issue arose: that of representations. While students may initially begin with informal understandings, they need to be inculcated into the discipline of mathematics, where there are established standards, procedures and vocabulary that enable those working mathematically to communicate understandings with each other. Students need to become familiar with these in order to authentically engage with others working within the discipline. The unit offered multiple opportunities to develop those understandings and one example is provided below which illustrates students' consideration of alternate data representations.

Table 6.7: Answers that reduced conviction

Limitation Recognised	Representation and Organisation	Interpretation and Analysis	Communication
Not enough information			X
Didn’t provide any evidence	X		
No open number line or a dot plot	X		
Messy – couldn’t read it	X		X
It didn’t show the ratio	X		X
A lot of unnecessary writing			X
People didn’t give an answer (a direct expressed answer)		X	X
Lack of explanation of answer		X	X

Table 6.8: Answers that were effective at convincing

Characteristics of ‘good’ responses	Representation and Organisation	Interpretation and Analysis	Communication
Making it clear what the ‘answer’ actually is			X
Making it clear how they got their answer (showing it clearly)	X		X
Had a dot plot (showed the results clearly)	X		
Used a range rather than a single answer (helped to see the cluster)	X	X	
Had really clear evidence (helped you to understand the answer and how the answer was arrived at)			X
Helped to see the data used (raw and organised)	X		
Writing was neat – at least able to be read.			X
Made it clear what you were measuring	X		
An explanation of why they came to the conclusion they did.		X	X

The students had suggested that tallies and dot plots could be appropriate representations for recording their data scores for the ‘normal’ human range. They had been debating the merits of both. However, the teacher ideally wanted the students to identify the potential of the dot plot to provide a good visual representation of a distribution. The students were beginning to learn to make statistical inferences and dot plots are more effective in displaying data characteristics than tallies: tallies do not incorporate equal intervals and thus can mask spread and irregular data. While it would have been far quicker and easier to simply instruct the students to make dot plots, the discussion below gave students the opportunity to see for themselves that different representations have different characteristics that influence their selection.

51.	Shana:	I think a tally is easier to understand
52.	T:	Well I'm going to get some data in a minute and put it up on the board and then maybe we can perhaps look at both and see what we think.
53.	Seth:	A tally is more (less?) work 'cause if you got like 2.7 and 2.4 [tails off]
54.	T:	So that means?
55.	Seth:	If you got like 2.7 and 2.4 then on a dot plot then you have to write like 2.4, 2.5, 2.6, 2.7, and 2.8.
56.	T:	So?
57.	Seth:	Well in a tally you only have to write down 2.7 and 2.4. A tally is more easier.
58.	T:	Easier to do what?
59.	Seth:	To organise the data.

The teacher approached the issue by planning to display identical data sets using the two representations for student comparison (52). Before she could begin, several students asserted that tallies are easier (51, 53). By eliciting the students' reasoning (53-59) a point was reached where there was an opportunity to challenge the students' thinking through class discussion focused on the purpose of the argument.

60.	T:	It is easier to organise the data. Is your aim to organise the data?...What is your aim?
61.	Connor:	To show the data.
62.	T:	To show the data? Is your aim to make it 'easy' for ourselves?
63.	Class:	No.

64.	T:	What is your aim?
65.	Ss:	To convince other people.
66.	T:	So can we go back and think about this from the point of view of convincing other people. What's going to convince other people? We talked about a cluster. What is going to convince other people that you have identified the cluster – a tally or a dot plot?
67.	Class:	dot plot, dot plot, tally, dot plot, dot plot, tally…
68.	T:	Why would a tally convince someone more than a dot plot?
69.	Shana:	No. Dot plot.
70.	T:	You want to change your answer to a dot plot? OK? [tone is questioning]
71.	Shana:	I want to change to a dot plot because you can like see the range through the dot plot with a tally you can't really see the range.
72.	T:	The range of…?
73.	Shana:	of the most popular like and like the outliers.
74.	T:	Aaah! So now we're talking about outliers. What will help you to see outliers better?
75.	Shana:	[The dot plot] because then you put in a space and it might be like the spots before and up to the number. Then just by looking at it you can tell like 'that can't be right'.

When one student changes her mind to a dot plot, the teacher accepts the response but makes it clear that she wants some sort of justification for the change (70); the students are not expected to change their mind because they think the teacher wants them to but rather because they can identify and articulate a reason to do so. One of the classroom norms for this class is that answers must always be justified, so the students were quite used to this as the class teachers routinely challenged correct answers as well as incorrect so that the students did not learn to 'read' the teacher but to provide considered reasoning.

<u>Third iteration</u>

After this discussion, the students elected to redo their posters, keeping the points noted in Tables 6.7 and 6.8 in mind. This would ensure that each proportion would have a range of what the students referred to as ‘normal’ human range with which to later compare Barbie. The students were instructed to go ahead and prepare their ‘B’ poster; however, this time they would be presenting them individually in front of the class and explaining them. The class would have the opportunity to ask questions and challenge claims and evidence (including interpretations). The purpose of this poster was two-fold. First it was hoped that the students would consider the lists derived from class discussion, and seek to use them to strengthen their evidentiary focus. Second, by shifting the communication from explanation (or rhetorical communication) to that which required persuasion (dialectical communication) (Berland & Reiser, 2009; van Eemeren et al., 1996), it was anticipated that students would need to rely on evidence more effectively in the knowledge that their explanation needed to be articulated effectively to others.

Once the students had compiled their ‘B’ posters, measurements were taken from Barbie and Barbie’s ratio for each proportion was noted. The students then considered their decision of a ‘normal’ human range, based on the dot plots they had provided as evidence, and made a claim as to whether or not Barbie could be considered human based on their data. This information was added to their B poster.

6.4.3 Interpretation and analysis of the evidence

The third epistemic marker, Interpretation and Analysis of Evidence, is perhaps one of the more difficult aspects for students in terms of the mathematics involved. In this criterion, students are required to apply mathematical content and procedure to making meaning of their results within a context. Often students need to go beyond the level of memorised knowledge to a deeper understanding of the mathematics in order to apply it effectively. The teacher had several target understandings she wanted the students to encounter. These included: the need for a range over a single modal score; the need to recognise anomalous data; and to consider the implications for this being sample data rather than population data. The excerpt below comes from relatively early in the Inquiry, taking place when the students had first returned their data, but before any attempt to organise it.

The teacher selected one ratio which has not been assigned, *length of hand: length of face* to use as a sample problem. A student, Gemma, made the observation that her adult had a 1:1 ratio for this score and tied that back to the unifix cube demonstration the teacher had previously conducted Section 6.3.6). In turn, the teacher showed the students how 1:1 could be represented mathematically, concretely (unifix cubes) and visually in context, with her own hand and face. The teacher then asks the students to indicate whether everyone had 1:1 and 'wonders' what should be done when they don't.

76.	T:	Did everyone get a 1 for that first one?
77.	Sts:	No (chorus)
78.	Salome:	I did though.
79.	Seth:	I did too.
80.	T:	What is this telling us about our idea of going and measuring Barbie and comparing?
81.	Shana:	We are going to have to do a range.
82.	T:	Explain.
83.	Shana:	Well. Maybe the most popular ones in the range and then we can see if Barbie is in the range.
84.	T:	What do you mean by the most popular ones?
85.	Shana:	The ones that have the most for girls.
86.	T:	So are we just going to look at the ones we have the most of?
87.	Shana:	A range from the least popular to the popularest.
88.	T:	OK. So how would we use that?
89.	Shana:	We can see if Barbie is in the range.

This discussion has helped the students to see the need for a range, that all scores will not be the same and that Barbie could fit anywhere within that range of scores and still be proportionate to a human. Shana's response (87) that the range would incorporate all data, does not yet address consideration of outliers, error, or the use of sample rather than population data. However, students are only just engaging with the data for the first time and these understandings can be (and were) addressed when the students have visual representations to aid them. The teacher had the students all provide the score they had collected for this measure and these were recorded on the whiteboard for the students.

The discussion offered potential for deeper exploration when one student provided a ratio of 0.1.

90.	Lee:	I have 0.1
91.	Sts:	Whoa!!!!
92.	Konrad:	How can you get that?
93.	T:	… What were your measures Lee?
94.	Oliver:	[quietly to Zachary]: Put your hand in front of your face. That’s like impossible!
95.	Zachary:	[Back to Oliver] But then your face would be about this big (holds finger and thumb approximately 2 centimetres apart)

Exploration of this score for raw data showed that the student had incorrectly calculated the ratio. However, this proved a useful opportunity to see how the students reacted to extreme scores and for the teacher to model how to check for errors. The teacher continued with the exploration of the sample data set, looking for opportunities to address other data issues the students were likely to encounter.

96.	T:	OK. What is the lowest score in the classroom for a female?
97.	Oliver:	0.9
98.	T:	What is the largest score in the classroom for a female?
99.	Seth:	1.2
100.	T:	OK. So we know that **every** human female face to hand ratio…we could measure **any** human female and we know that the ratio will be somewhere between 0.9 and 1.2?
101.	Sts:	No!
102.	T:	Hey?? [feigning confusion]
103.	Konrad:	The more people there is, like for the range, the better it is.
104.	T:	Why?
105.	Konrad:	Mmm. ‘cause Barbie [pause] ‘cause another person, her face could be higher than 1.2.
106.	T:	So you’re saying we could go and measure someone who is not a parent in this class and their face could be say 1.3?
107.	Konrad:	Yep.
108.	T:	Or 2.7?
109.	Konrad:	Yeh.

110.	T:	What about 10.9?
111.	Sts:	NO!!!
112.	Delmar:	No way.
113.	Konrad:	Impossible!!

The use of extreme examples while leading the students through this activity indicated to the teacher that the students had assimilated the understanding that values would have to be limited to an extent by the context. One advantage of the use of extreme examples is that it both demonstrated the ridiculous to the students and thus effectively demonstrated the point, but it also had the advantage of helping the teacher to quickly assess for developing knowledge; if the students hadn't reacted to the extreme, it was unlikely they were conceptualising the mathematics in terms of the context.

With the teacher satisfied that the students had at least a rudimentary appreciation for the concepts necessary to continue, she encouraged them to redesign their responses to "*What is the human proportion for the ratio you have?".* Students' responses were collected for assessment and then returned to them after individual conferencing with the teacher. The final task was to make a claim as to whether Barbie's ratio could be considered to be consistent with that of a 'normal' human and to prepare a presentation to communicate to, and convince the class, they had drawn appropriate conclusions. This task enabled the students to make the transition from preparing an argument product to presenting the argument as a process.

6.5 Argument as a Process – Communicating the Argument

Argument has at least two meanings, and while it has previously been discussed as a product, it is also a process, with a defined purpose and generic norms. In both cases, these generic criteria are in part determined epistemically. Thus students need to learn not only the requirements for the structure of the argument but also what is valued in terms of arguing the position within a specific discipline. Students also need to learn to construct an argument and to present it in both written and oral form. It is in the presentation of the argument to others (whether written or oral) that the audience can be persuaded to some belief or action (the goal of argumentation); including the action of proposing or defending an alternate position. In this section, the students' first experiences of engaging in and learning to challenge each other's arguments are discussed.

At this point, the students had completed their 'B' posters, had determined a likely range for 'normal' humans and had been encouraged to construct an oral presentation (with a supporting poster) which would convince others of whether Barbie had human proportion for the measurement they were looking at. It was stressed that the students would be allowed to question and challenge each other's evidence (respectfully) and their decision as to whether Barbie had human proportions. Effectively they were being positioned in the level of dialectical communication (van Eemeren et al., 1996); that which aims to persuade (Berland & Reiser, 2009).

At the conclusion of each presentation, the teacher asked the class to question the presenter and then she added questions of her own when the students had finished. Initially, the students displayed reticence and so the teacher modelled questions intended to elicit understandings for the first few presentations. By doing so it was intended that the nature of the dialogue would make the teacher's cognitive processes more public to the students. Connor's presentation below, which was one of the first, shows sample questioning by the teacher. For brevity, approximately 16 lines have been removed, however; they are all student- teacher interaction of the same nature as below.

114.	Connor:	My question is: Does Barbie have the same proportion as a human? I found out that Barbie does have the same proportion as a human from my proportion which was shoulders to hips: top of head to shoulders. Barbie's ratio which was 1.7, fits within the [pauses] between the normal human range which was 1.6 to 1.8. I found out Barbie's ratio by dividing her shoulders to hips by her top of head to shoulders. The end.
115.	T:	You said that you had a human ratio of 1.6 to 1.8. Can you tell me how you found that?
116.	Connor:	By finding all the data and putting it into a dot plot and looking at where the cluster was, which 1.6 is. I got the data and found out which place has the most frequent score and that data was from 1.6 to 1.8.

117. T: So all of your data were between 1.6 and 1.8?

118. Connor: No, the most frequent data was that.

119. T: What other data did you have? Because I can't see that.

120. Connor: 4.2 and 28.8 which were giant outliers.

121. T: So how do you explain those two giant outliers?

122. Connor: They are probably both wrong. Both been measured wrong.

By the fourth or fifth presentation, the students began to adopt the questions being asked by the teacher and were tentatively using these in forming their own questions. Before long, the teacher was able to step back as the students took over questioning their classmates. In the excerpt below (18th presentation - Geneva), the teacher had very little involvement until the students had exhausted their questions and, based on the information elicited by the students, the teacher saw an opportunity for deeper exploration of Geneva’s understandings. Geneva’s actual data sample has been reproduced in Figure 6.4 below to enable the reader to more easily follow the conversation.

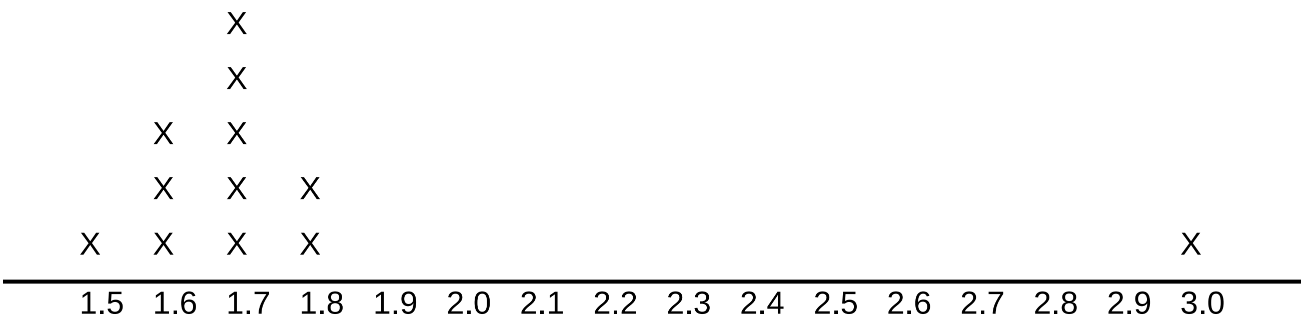


Figure 6.4: Distribution of Geneva's sample data

123. Geneva: I think Barbie is a human. Wait. My question is, “Has Barbie got human proportions?”. My answer is yes. This is why. I think Barbie is a human because her ratio 1.7, is the most common human ratio. My evidence that her ratio is 1.7 is, 1.7 divided by, I mean, 17 divided by 29 equals 1.7 (sic: 29/17 = 1.7). Evidence that the human ratio is 1.7: Here is my dot plot. There is one 1.5, three 1.6’s, five 1.7s, two 1.8s, and one 3.0. 3.0 is the outlier. So I think Barbie’s ratio is a human ratio.

124. Oliver: What were you measuring?

125. Geneva: Navel to foot divided by height (sic).

126. Delmar: Did you check, did you go back and check how you got the outlier, or have you not done that?

127. Geneva: I haven’t done that yet. Salome?

128. Salome: Do you have a range on your dot plot?

129. Geneva: The range is, well the most, I said the normal range could be from 1.0 to 3.5, but I’m not completely sure about the 3.5 part. Because 3.0 was the outlier.

130. Seth: Um, so if Barbie, what’s your range again?

131. Geneva: My normal range? From 1.0 to 3.5, but I’m not completely sure about the 3.5 because 3.0 was a very big outlier compared to the others.

132. Oliver: If Barbie was 3.6, would she be not normal?

133. Geneva: I’d say so, because then she’d be 0.5 over the outlier. Shana?

134. Shana: Do you know how you got the outlier?

135. Geneva: [Shakes head] No.

As can be seen, the questions asked by the students were very similar to those initially modelled by the teacher and it would appear that the students identified and adopted or mimicked the teacher’s ideas as to what was important to question about the data. The teacher, and another researcher who was present that day (R in the transcript below), were then able to extend Geneva’s thinking by posing further questions. The excerpt below highlights the extent to which the students’ thinking can be explored through questioning and also the possibilities for exploring and challenging student understandings to bring about that doubt that is necessary to lead students towards disequilibrium. While this dialogue continued between the teacher, researcher and Geneva, the class was listening attentively and developing their own understandings. Ultimately, it was a focus on the evidence in context which enabled this issue to be addressed. Not only did Geneva bring her attention to the evidence, but ultimately to the correctness, or quality of that evidence.

136. R: You’re pretty sure about the 1.0.

137. Geneva: Sort of. Pretty sure.

138.	R:	So, sort of pretty sure, what do you mean?
139.	Geneva:	Well, I’m not completely sure, but I’m pretty sure it could go down near 1.0. So I just wrote that.
140.	R:	So what would have to happen for it to go down to that?
141.	Geneva:	Maybe someone was quite small but um, their navel to foot made up most of their height, or something like that.
142.	T:	Ok, Geneva, one more question for you, um, you identified an outlier of 3. What do you think a person would look like if they had a ratio of 1:3? So what do you think a person would look like, you said you did navel to foot, to height. If a person had a ratio of 3, what might they look like? Could you visualise that person?
143.	Oscar:	Holy moley!
144.	Liam:	Holy!
145.	Delmar:	That's impossible
146.	Geneva:	Maybe they, um, maybe they were like, they could be like really tall, but not have very long legs, or something like that?
147.	T:	Ok. So let me [T stands up]. … you could use me as an example. Navel to foot [indicates on herself the distance from her navel to her foot], that would be 1, and my height would be 3 times that [indicates the distance that would be 3 times the navel to foot]. If I had a ratio of 1:3, navel to foot, my height would be three times that. So here’s my navel. Here’s my foot. [Draws a stick figure on the board]. And my overall height would be one, two, three [draws].
148.	Geneva:	They’d have to have very short legs, though. …
149.	T:	What would you do now? If you are thinking that this isn't possible or that you are not certain that this is possible, what could you do now?
150.	Geneva:	Check out the person measured at 3.0 and make sure they measured it correctly.
151.	T:	Anything else?

152. Geneva: Make sure there wasn't something with the person; they didn't have an injury or something.

The other important feature of the second excerpt (136-152) is that it was the first significant instance of a student attempting to defend their evidence and reasoning. Prior to this it was noted that if a teacher challenged the students reasoning, the students would often assume it to be incorrect without thinking it through. The boys above reacted quickly to Geneva's comment with incredulity (143-145) without thinking through the possibilities as Geneva proceeds to do. While one of those boys, Delmar, had previously demonstrated the ability to rapidly visualise proportions, it was highly unlikely that the other two were doing more than responding to the teacher's challenge. The idea that teachers challenge incorrect answers and simply accept those that are correct was clearly ingrained in these students and it took significant work by the class teachers to break this perception: including repeatedly and explicitly advising students that they would challenge reasoning regardless of its 'correctness'.

Throughout the unit, the students had opportunities to work with individual data samples, to draw conclusions and orally present and defend their conclusions about the likelihood that Barbie would fit into the population from which the sample was drawn. It should be noted that there were differences between the data sets in terms of whether Barbie's proportion was clearly within the sample, a great distance from the sample, or on the cusp. The former two instances were clearly easier for the students to make a claim against and support in terms of reasoning. However, as students presented their findings to the class, all students were involved in the discussions surrounding these issues and were able to engage with the exploration of various samples.

6.6 Assessment of Argument Product and Process

For research purposes, evidence of students' progress was documented to monitor them in terms of both the content addressed in this unit and students' developing knowledge of argumentation product and process. This evidence served to evaluate the impact of teaching and learning experiences across multiple factors, and to identify areas that would need addressing during the next iteration of the research. This section describes the final assessment task given to the students which was completed individually, and discusses observable changes across the course of the unit.

After each student had presented their findings and had an opportunity to be challenged (see Section 6.5), the students were provided with a culminating assessment task. The nature of this final task was for the students to work with a raw data set for ratios not previously seen by them (length of forearm: length of hand), to ascertain individual development. The actual data set provided to the students is in Figure 6.5; however, the data the students received was in a raw, unorganised format. This data set was specifically selected as it contained two obvious outliers, a lone piece of data in close proximity to the central clump, and a ratio for Barbie that sat outside the clump but not excessively so (giving opportunity for reasoning). Observing the data here, a definite clump towards the centre of the range can also be noted. If more data were obtained, it would be conceivable, based on a visual assessment, that data could easily range from around 0.9 to 1.7 or further. Certainly, 1.0 could not be excluded as erroneous or outlying data. The 0.2 and 2.4 scores are actually erroneous and a check of the calculations performed by the students gathering these data would indicate that if the students checked. This data set provided opportunities for students to reason about peripheral data and so enabled exploration of the depth of students’ understanding and the epistemic acceptability of their reasoning.

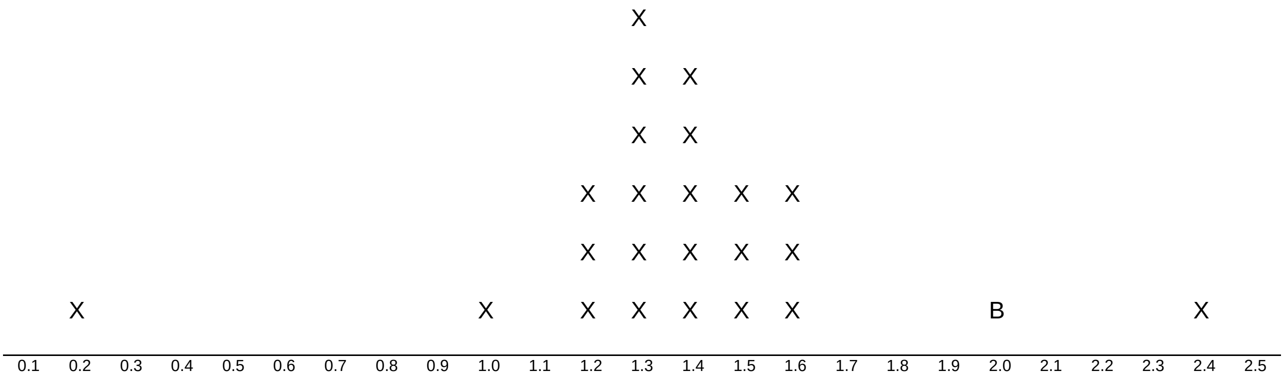


Figure 6.5: Data set used for the culminating task (length of forearm: length of hand), with Barbie’s ratio (B) indicated

What are determined to be epistemically acceptable responses amongst the community of learners must reflect what is accepted by the discipline. In this study, the aim was for the students to make successive approximations towards the norms of the mature discipline. So ideally the community of 8-9 year olds would begin to make informal inferences; to recognise that this is a sample and from the sample we can infer what the population might look like. Thus important indicators in student reasoning would be those that show that students can see the human populations will not be identical to the sample they have, that the population data will likely have greater spread; but not too much greater, as their proportional reasoning context should inform them that there is a limit to what is reasonable. A sample of work is displayed and discussed below to enable illustration of one student's responses.

The solution sample displayed below from Denise (Figure 6.6) suggests she was demonstrating a level of statistical reasoning that was quite strong at this stage. In comparing Barbie's forearm: hand ratio to a human's, Denise has represented the 'normal' human range as extending slightly beyond the existing data (1.0 – 1.6) which suggests she may be aware these data are a sample only and that there are other members of the population whose data have not been included.

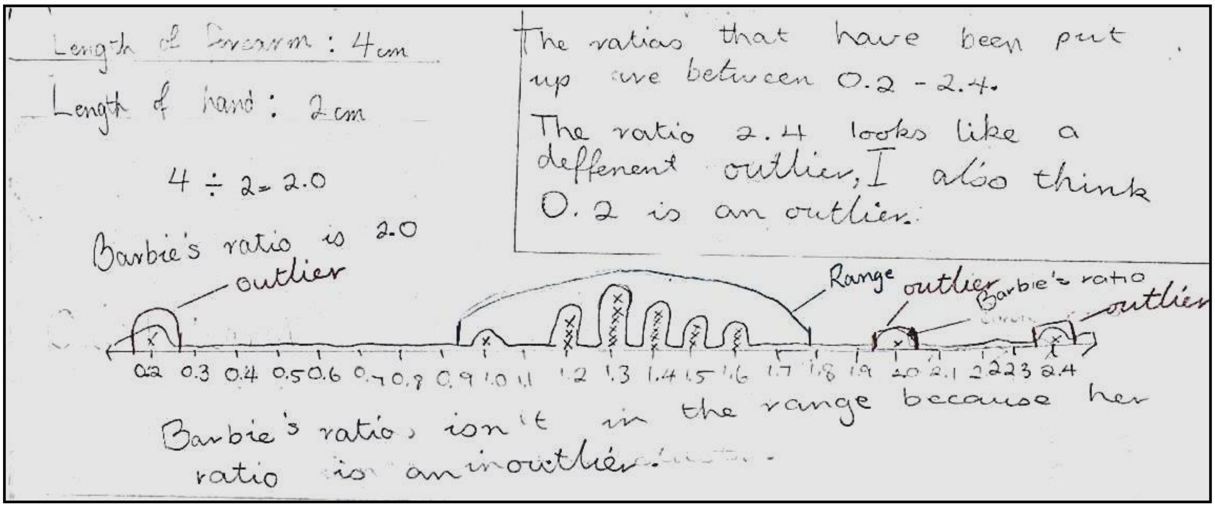


Figure 6.6: Sample student response to the culminating task (Denise)

6.7 Assessment of Development over the Course of the Unit

Assessment of the students' work over the course of the unit was undertaken using the criteria standards located in Section 5.7.2. The criteria are organised under two broader categories, Argument Structure and Epistemic Reference. The primary differences between categories are based firstly on the presence of the criteria component (scores around 1 and 2 are problematic), the completeness of the component (a score of 3 generally means the component exists but is insufficient in some respect), and the extent to which the component approaches ideal for this community (scores of around 4 and 5). Overall, using these criteria to examine students' work was shown to be quite effective in terms of identifying variation in student understandings and development. By using such criteria, it becomes possible to quantify shifts in student responses.

The focus of the Barbie unit was not on Argument Structure, but rather on establishing an initial focus on evidence and on beginning to consider what constituted acceptable evidence (Epistemic References) in our relatively novice mathematical community. Argument Structure essentially comprises the component parts of the argument itself; whether there was a claim, whether evidence was provided, the extent to which evidence co-ordinated with the claim, and so forth. Students adequately provided these components for the most part as they had been supported to do so. Therefore, any shift in the scores for the first category, Argument Structure, was largely a result of incidental learning. However, in the next research iteration (Chapter 7), argument structure is addressed more formally and so initial scores are reported here (Table 6.9: greyed out) to establish a baseline for later results.

In this initial inquiry, the Epistemic Reference criterion was more relevant in terms of assessment. The focus was on ensuring students could see the requirement for mathematical evidence and the acceptability of the evidence in terms of the mathematical discipline, at least in terms of this specific community of learners. It is within this section that any gains in mathematical understanding would be reflected.

The first work sample chosen for full analysis (n=25) was the students' initial response to the question about what was 'normal' for humans for the score they were working with (their 'A' poster, for an explanation of this activity, refer to 'second iteration' in section 6.4.2). An example of the progress of a sample, middle-of-the-road response to this task has been provided for illustration. Oliver's initial attempt at addressing this question is included at Figure 6.7.

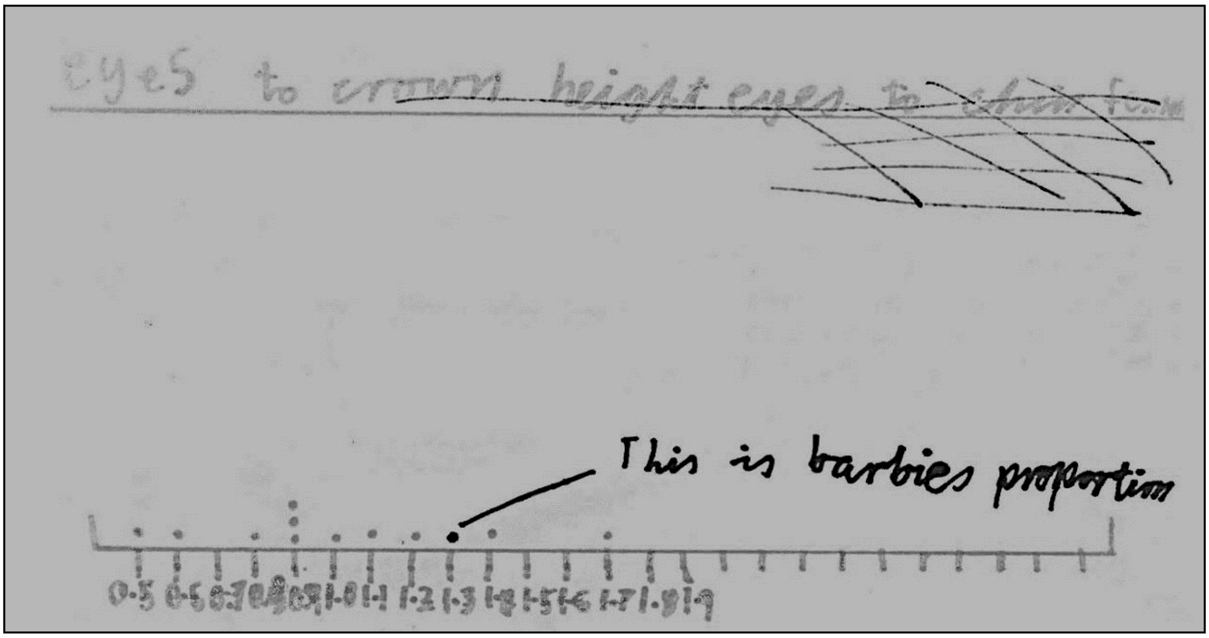


Figure 6.7: Oliver's early attempt to respond to the question (eyes to crown : eyes to chin)

The culminating task (n=25) was their final, individual task in the unit which all students attempted as a summative activity. Oliver's response to this final task has been included below at Figure 6.8 to enable illustration of the criteria and to give the reader a sense of the student's thinking.

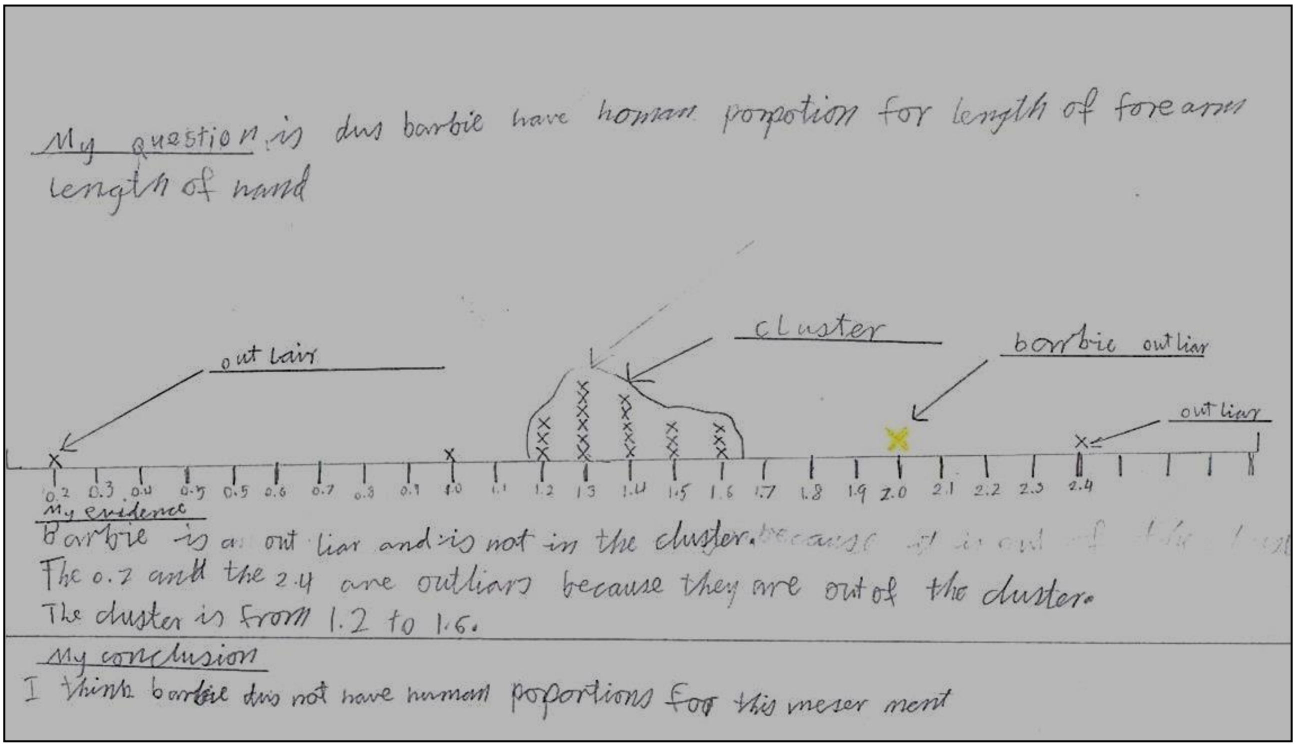


Figure 6.8: Oliver's final response - comparative task (length of forearm: length of hand)

Overall, students were largely capable of providing evidence derived from data gathering, organisation and analysis. The large increase in scores in Evidence Organisation (Item 11, Table 6.9) resulted from students learning conventions for creating data representations, particularly dot plots. Consider Oliver's dot plot (Figure 6.8): the distribution is clearly identified; the intervals are even and complete (no values missed); all of the data have been included; each dot represents one value for the variable of interest; and the column height clearly and accurately demonstrates the frequency. There was an absence of the customary labelling of the axis which is usual with graphical representations in mathematics; however, when considering the audience (the class) the students and teacher were well aware of what the graphs represented so the influence of audience could well have dictated here.

Interpretation of the dot plots (Item 12, Table 6.9) was generally well done with virtually all students providing a plausible suggested range for human proportion. As can be seen from Oliver's solution, the score of 1.0 was problematic; several students, including Oliver, did not include it in their cluster or identification of a 'normal' range. Having a score this close to a significant cluster invites the acknowledgement that increasing the sample size would potentially 'fill the gap'. While several students indicated awareness that they were working with samples (as distinct from populations) by incorporating this score, this was not universal. Previous research findings suggest that it can take time to develop an appreciation of the differences between population and sample (Pratt, Johnston-Wilder, Ainley, & Mason, 2008).

The final category is that of reasoning (Item 14, Table 6.9), which is essentially the justification for making the claim based on the evidence. Oliver argues that Barbie *does not have human proportions for this measurement* because *she is an outlier and is not in the cluster.* In terms of statistical reasoning, this can be deemed acceptable in terms of Oliver's expected level of understanding. Ideally the student would make some reference to the proportional distance the score of interest is from the main cluster, as well as observe the spread of the data as these are impacting factors on measuring deviation.

Table 6.9: Comparative scores from 'A' poster to culminating task

	Indicator	Descriptor Within Community Standards:	Initial Response (Poster A) n=25	Comparative Response n=25
	Argument Structure			
1	Research Question	The research question is clearly and specifically stated	a*	2.5
2	Research Question - Context*	The research question informs the wider research context	a*	a*
3	Claim	The claim is explicit, foregrounded and references the question	a*	2.4
4	Evidence (Grounds)	Evidence reflects audience, is relevant to the research question, contains sufficient (but not extraneous) detail	3.5 (c*)	3.8
5	Reasoning	Reasoning co-ordinates logically and considers all evidence	2.9	3.0
6	Claim – Context*	Claim implications for wider context are explicit	b*	2.2
7	Qualification	Qualifier is provided with details as to when it is applicable	1.0	1.9
	Epistemic Reference			
8	Evidence Collection	Evidence collected / generated responds to the question being asked	a*	a*
9	Foundation for the Evidence	Evidence provided is data-based (as distinct from fallacy, conjecture, opinion)	a*	a*
10	Evidence Gathering	Methodology for obtaining evidence is provided and is appropriate	a*	a*
11	Evidence Organisation / Representation	Representation/ organisation of data are accurate and appropriate for the audience and purpose	1.9	4.3
12	Evidence Interpretation / Analysis	Interpretation / Analysis of evidence meets community expectations: accuracy, clarity, method, efficiency	3.4	3.7
13	Evidence Anomalies or Contradictions	Any anomalous or contradictory evidence is provided and addressed factually or in terms of limitations	3.0 (d*)	3.3
14	Reasoning	The justification for making a claim, based on the evidence, is suitable given the community of mathematical learners	2.4	3.0

a* - information that was provided by the teacher and therefore not assessed
b* - no wider context was applicable to this question
c* - significant guidance through in-class discussion and collaborative planning
d* - these results may be misleading as the students could only be assessed on this if their data set included anomalous or contradictory data and in the first instance many did not and these results were derived from n=9 - the culminating activity included anomalous data, n=26

6.8 **Summary**

This chapter presented results around the first iteration in the research design; a mathematical inquiry in which students sought to respond to the question, “Does Barbie have human proportions?”. Results provided were organised under the wider considerations of ‘argument as a product’ and ‘argument as a process’. Considering the argument product, the chapter identified some of the struggles students encountered, including: the recognition of the need for evidence, envisaging and planning for evidence gathering, and organising and interpreting evidence. In terms of argument as a process, the nature of the engagement of the students in presenting their arguments has been illustrated. These students began developing an appreciation for the discourse in argumentation by modelling appropriate questioning from the teacher initially. Finally an explanation of the research ‘assessment’ was addressed. The assessment was undertaken using a framework devised from student arguments, and indicated a forward shift in students demonstrating knowledge of what is epistemically acceptable in mathematics learning. In short, this chapter has introduced the use of argumentation as a pedagogical tool in the classroom.

In the next chapter, the results of a second unit are addressed. This unit was undertaken by the students some six months later, and took a specific focus on introducing argument structure to the students more formally. Students considered claim, evidence, reasoning and qualification through a mathematically-contextualised research question, “*Can a pyramid have a scalene face?*”. The focus on evidence continued as students addressed issues of quality and quantity of evidence.

7 Results – Intervention 2

7.1 Chapter Overview

The unit described in this second results chapter commenced some eight months after the completion of the Barbie Unit, and midway through Year 5. The unit was built upon the inquiry question, "*Can a pyramid have a scalene face?".* While the content of the topic is clearly geometrical, the argumentation goal of the unit was to formalise and extend argument structure with the students. Several reasons existed for the delay between units; the first being a natural disaster earlier in the year which had significantly damaged the school and which required the reconstruction of this class' building. The teacher preferred to 'normalise' the situation and wait for the return to a proper teaching space (they were working in a very cramped temporary room with limited facilities and resources for 4 months). The second reason was the involvement of these students in the mandated national testing program (NAPLAN – National Assessment Program: Literacy and Numeracy), a high-stakes test which every Year 5 class in the country undertakes in May of each year. Hence, it was the middle of the school year (the Australian school year runs from January to December) before teaching and learning settled back to normal. The involvement of the students in NAPLAN became a potentially confounding issue as the Writing test item the students were required to be prepared for was a persuasive text. Accordingly the class had been working extensively on persuasive genre and had learnt a 'model formula' - claim, three 'arguments', conclusion - along with the use of persuasive devices which included the use of emotive language. It was important that the students saw that a far different form of argument is required in mathematics and that the discipline relies on objective, testable theories; in short, mathematically defensible evidence. Thus this unit introduced both evidence quality and deepened argument structure.

As was the case in the previous results chapter, this chapter has been organised primarily into two strands: 'argument as product' and 'argument as process'. The chapter commences with students reviewing the Inquiry Model and expanding 'Conclusion' to incorporate an Argument Model, before engaging in the inquiry content. There is a focus on Argument as Product which addresses the students' progression from their own initial understandings, to their articulation as they work together in groups, and finally to their engagement with persuasion as they seek to convince their classmates of the value of their evidence. Evaluations of the students' initial and final arguments are included and discussed at the conclusion of the chapter, along with an overview of the students' development across the course of the entire study.

7.2 Argument as a Product

At the commencement of this unit, the students revisited the evidence model as they left it in the Barbie unit. They had largely maintained their familiarity with the model and class discussion was used to develop the model further to incorporate aspects of argumentation more formally. In the Barbie unit, the students were unaware of the component parts of an argument; however, the purpose of undertaking this second unit was to address the structure of an argument and have students consider the components of claim, evidence and reasoning.

7.2.1 Introducing the inquiry question

When working with inquiry, the classroom teacher in this study was accustomed to introducing an ambiguous, ill-structured, contextualised question and using the refining of the question as a means to both engage the students and to have students explore the mathematics that would likely arise. This unit deviated from that pattern in that the students had initiated an inquiry question while undertaking a lesson on pyramids, asking if they could carry out an inquiry to find out: "*Can a pyramid have a scalene face?*". The teacher decided that the opportunity to have students explore mathematics they were interested in was sufficient reason to address the problem and was unwilling to rebuff the interest and initiative demonstrated by the students in developing their own research question. Thus this unit was grounded in a solely mathematical context.

7.2.2 Expanding the evidence model

In order to review prior learning, the unit commenced by having a few students construct the Evidence Model (Figure 7.1) on the whiteboard using pre-prepared magnetic labels. Even though the students demonstrated familiarity with the model, the teacher quickly reviewed components and revisited the argument terminology they had been previously introduced to (claim and evidence), before focussing on the question.

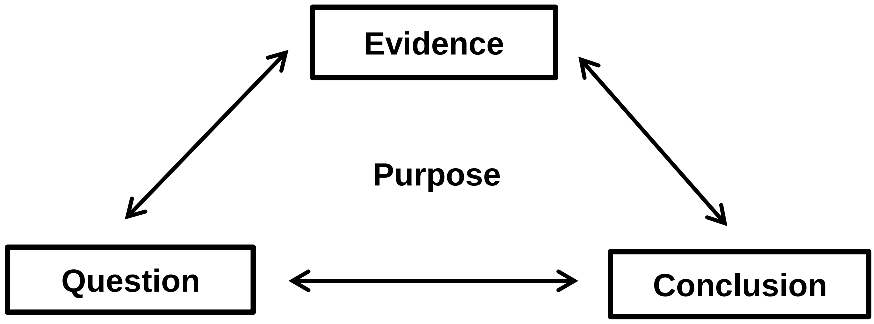


Figure 7.1: Evidence Model

In the teacher's approach to introducing the unit, she made it clear from the outset that the students were focusing on the conclusion aspects of the model. As the students were largely familiar with the mathematics underpinning this unit, the teacher had decided this was an opportune time to deepen their understanding of the structure of an argument. This was based on a premise that the familiarity with the mathematics would enable students to envisage the evidence they would need to gather and work with: that the familiarity of the mathematics would serve to scaffold the argument. This is in contrast to the Barbie unit, where familiarity with the context was used to scaffold the mathematical understandings and argumentation product and process. The class began by expanding the Evidence Model in light of their previous work with Barbie, and did so by incorporating aspects of the Argument Structure; specifically 'claim' and 'evidence' were added to the conclusion (Figure 7.2) while being discussed in some detail.

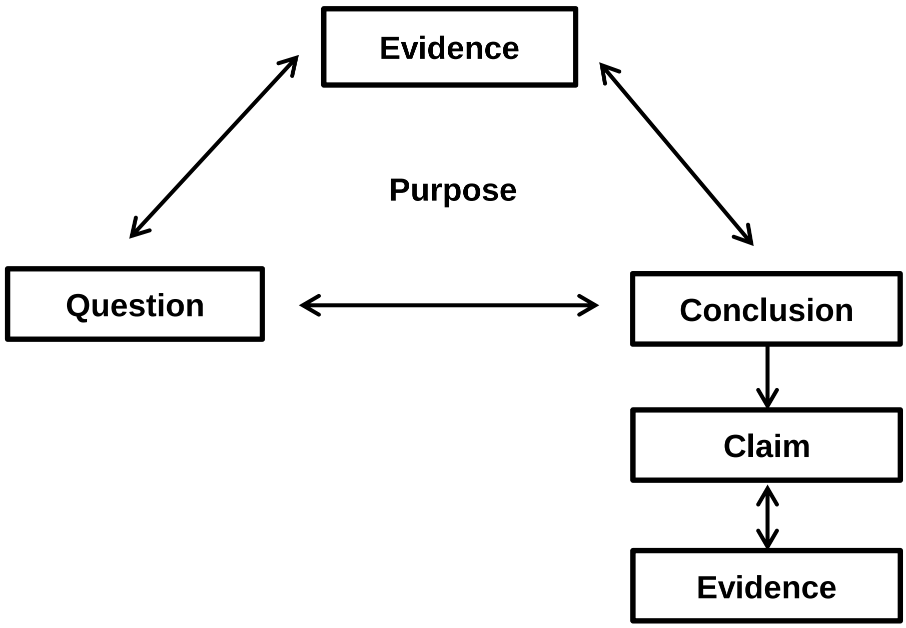


Figure 7.2: Evidence model expanded with argument components

1.	T:	This evidence cycle has been growing since you guys were in Year 4. There is something I want to add today that you haven't seen before [though they had been exposed to the terms in the prior unit]; this is part of the conclusion. Now a minute ago you said conclusion has to link to what?
2.	Zachary:	It has to link to all of them [evidence, question and purpose].
3.	T:	So when I make a conclusion, when I decide whether or not a pyramid can have a face which is scalene, what's going to be in my conclusion?
4.	Connor:	A claim
5.	T:	Uh huh - a claim. So you saw it before. So what's a claim?
6.	Connor:	Something that you....[tailed off]
7.	T:	Think about the persuasive arguments that you have been writing for Mrs F [the co-teacher on the class].
8.	Connor:	Something that you say and then you give evidence.
9.	T:	Ah. So we have a claim which is something that we believe, something that we think, something that we have concluded from our inquiry. So we have a claim. From our claim, what was the next thing that you said? [pause then prompting] From your claim you have to have?
10.	Connor:	Evidence
11.	T:	Evidence. Why do I have to have evidence? Why can I just not say "Can a pyramid have a face which is scalene? Yes!" Ahh...no someone different - Konrad?
12.	Konrad:	How can we believe you?
13.	T:	How can you believe me? [Acting exasperated] How can you believe me? I am a teacher!
14.	Konrad:	Teachers aren't always right.

The exchange above draws out two important responses from the students. The first is that they had recalled the need for a claim and evidence to support that claim, which was the purpose of the teacher's review. The second was Konrad's response that "teachers aren't always right” (12-14) that suggested that the students were beginning to shift towards a learning community as distinct from seeing the teacher as the source of knowledge. To have students see themselves as part of a knowledge building community was an important goal.

In the first unit, the students were introduced to the idea that evidence is required to support a claim. In this unit, the purpose of evidence, and what constitutes acceptable evidence was addressed.

15.	T:	OK so you need the evidence so that people believe you. OK. So you need the evidence to do what?
16.	Lucy:	To convince
17.	Connor:	To persuade
18.	Shana:	With evidence you can back up, support your conclusion.
19.	Oliver:	You have to have evidence because a claim is like trying to get someone to believe what you are saying you make it stronger by giving it more evidence. You give it evidence which will make your claim better. You can make your conclusion better.
20.	T:	So to strengthen it.
21.	Connor:	Yeah, it’s to add detail.
22.	T:	So my claim is basically my answer isn't it?
23.	S's:	Yeah
24.	T:	So let's try this. I haven't actually done it [answered the question] so let’s just say I discover a pyramid can't have a scalene face. So my claim is 'No. A pyramid cannot have a scalene face'. That is my claim. My evidence backs up, supports, aims to persuade, aims to convince [pointing to each child who had previously given those answers as reasons for using evidence in your conclusion] people that my claim is correct. That my claim is…
25.	Lucy:	[interrupting] true.
26.	T:	True - thanks Lucy. So my evidence helps to show the truth in my claim. We would say the validity; it shows how valid my claim is.

In order to highlight the roles of claim and evidence and to have students begin to consider the nature of the evidence, the teacher used a simple, intentionally humorous argument to engage students and to help the idea to ‘stick’. This argument ended up being replayed several times through the course of the unit as a demonstration argument. Over the ensuing conversation it became clear that the students were fully aware that mathematical arguments did not rest on persuasion or opinion.

27.	T:	OK. What do you think of this? 'Mrs W is the most gorgeous teacher in the universe'. My evidence is I had a look in the mirror this morning.
28.	S's:	[Sniggers]
29.	T:	How strong is my claim?
30.	S's:	Not very (simultaneous)!
31.	T:	Why is my evidence not very strong for my claim?
32.	Lucy:	Because you said that. You don't have anybody else who said that.
33.	T:	So you're saying there is something missing - can have evidence but the evidence might not be very strong.
34.	Kody:	It might not be very strong but if you add more it might get stronger.
35.	T:	So my evidence might not be very strong. I might not have enough evidence.
36.	Connor:	Your evidence doesn't really say you are, it just says your opinion.
37.	T:	So if something is just an opinion and I don't have a lot of support for it, does that make a strong argument?
38.	Connor:	You need the majority of the Earth to say it.
39.	T:	OK - so Laverna thinks I am the most gorgeous teacher in the universe. That's two of us. Do I have enough evidence now?
40.	S's:	[chorus] No!!
41.	T:	My husband thinks I am - that's three.
42.	Connor:	You have said three people have but that's only three. You haven't said how many have said no.

43. S’s:	It’s not true	
44. T:	So we say that the evidence isn't valid.	
45. Oliver:	And your claim has to link to your question - it has to answer the question.	
46. T:	So does the claim lead to the evidence?	
47. Shana:	No - the evidence leads to the claim.	

The conceptual understanding demonstrated by Shana (47) essentially reflects the difference between inquiry argument and advocative argument and is an essential distinction for students to be aware of when working with the same genre (argument) across multiple disciplines. In the sciences, including mathematics, questions are addressed by gathering evidence, analysing the evidence and then drawing conclusions or making a claim (inquiry argument) (Toulmin et al., 1984). However, in the persuasive text types usually addressed in English lesson, students are often required to select a position or claim, and then defend it by selecting supporting evidence and even obfuscating contradictory evidence (advocative argument). The disciplines of science and mathematics would regard this selective approach to evidence as unacceptable.

The students subsequently compiled an informal list of what they felt to be the collective requirements of mathematical evidence. According to the students, the evidence should: be true, support the claim, convince, back up the claim, be strong, persuade, be relevant, be accurate, and make sense. As the students were clear on the nature of inquiry evidence, the teacher moved on to the concept of ‘reasoning’.

48. T: So is there something else we need to add here [to the Inquiry Model] that kind of links the evidence and the claim and tells us how good the link between the evidence and the claim is?

49. Konrad: Like the reason

50. T: OK - so where do we put reasoning?

The students reorganise the diagram to incorporate reasoning as well in the form of a three-way triangle with double ended arrows under conclusion (see Figure 7.3).

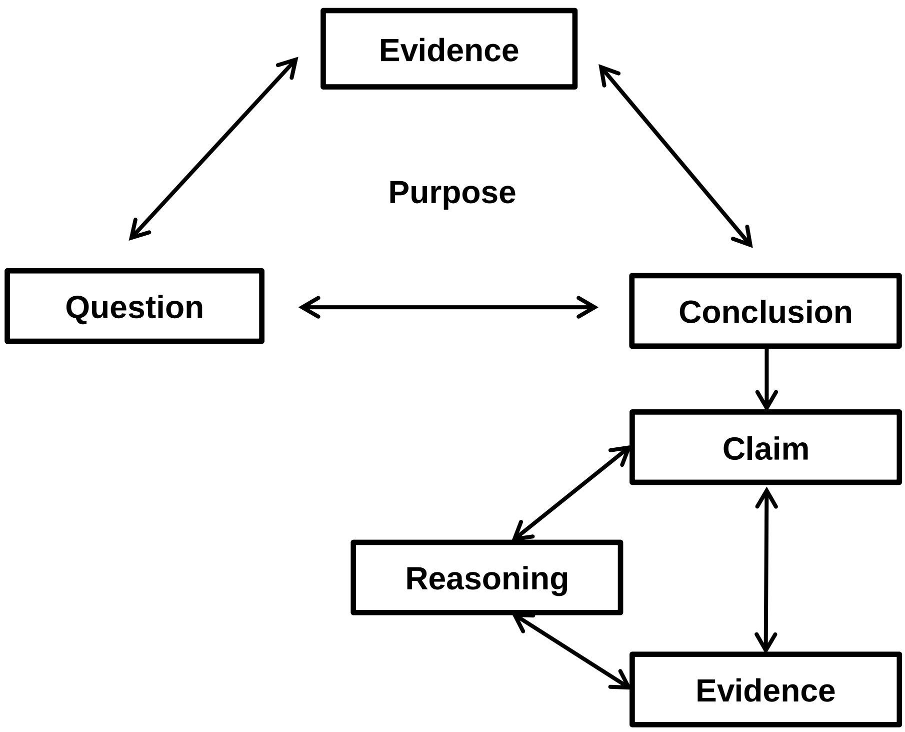


Figure 7.3: Incorporating reasoning into the conclusion

51.	T:	So this [indicating] your claim, your evidence and your reasoning goes together to make up your…?
52.	S's:	Conclusion
53.	T:	Conclusion. And your reasoning is really linking your evidence to your claim.

Once the students had considered the components of a conclusion, which were essentially the components of an argument, they had some understanding of what was to be the focus of the inquiry and this enabled the students to consider what might constitute evidence in this inquiry. The next session addresses the aspects of argument as a product. The students commenced by considering the evidence required and planned how to obtain it.

7.2.3 Envisaging and planning to obtain evidence

The students were asked to envisage the evidence that might assist with addressing the question; however, they did not have the support given in the previous unit on Barbie (in which the students were scaffolded to envisage the evidence by envisioning possible claims and supporting evidence). One reason for this was that the students were more likely to be able to envisage the evidence required as the mathematics involved was a familiar aspect of geometry. A second reason was that the students' inquiry did not encompass a context outside of mathematics, thus it was unlikely that context knowledge would interact with the mathematics.

As the students were considered to have the necessary mathematical understanding, they moved straight into working together as groups to consider the evidence they would need. The excerpt below, illustrated one group [Salome, Sadie, Lee, Geneva] as they worked towards envisaging their evidence:

54.	Salome:	So what would count as evidence?
55.	Geneva:	A model could. If you get a model with at least one scalene side then it would be evidence because obviously it would be possible.
56.	Lee:	Maybe a diagram.
57.	Sadie:	A model because it actually does show us.
58.	Salome:	[talking aloud as she writes] A model of a pyramid with one face that is scalene. And I like Lee's idea about a diagram. A diagram of a pyramid. And a diagram of a pyramid with one face that is scalene.
59.	Geneva:	A net
60.	Salome:	But isn't the net the diagram?
61.	Geneva:	A measurement - a measurement on the diagram.
62.	Lee:	A net of a scalene pyramid.
63.	Salome:	A net might not be great. We might need to test it.
64.	Sadie:	Yeah. Test it because it might be wrong.
65.	Lee:	A testable net.
66.	Salome:	No, an already tested net.
67.	Geneva:	A correct net.
68.	Salome:	[out loud as she writes] a correct net of a pyramid with one face that is scalene.
69.	Sadie:	Shouldn't we say **a** [emphasised] face 'cause one face might not fit with...[tries to indicate the interlocking with the touching faces but can't find the words].

70. Geneva:		So what evidence could you have that a pyramid cannot have a scalene face?

As was expected, the evidence envisaged by the students was mathematically focussed and this was true of each group collectively (as the students presented their information as a group it was not possible to determine individual input in this instance). While the group could demonstrate an evidentiary focus that would be practicable and would address the question, they did suggest a quantity of evidence which would have been far more than necessary. In particular, they focussed on providing multiple representations of similar information: a model (55, 57), a diagram (56, 58, 61), and a net (59, 62, 65, 66, 67). This desire to provide *quantity* of evidence was evident in the only other small group captured on video (Lucy, Shana, Dominica), although it was a recurrent theme through the other groups' presentations held later.

This desire to provide a quantity of evidence may be explained as a response to the perceived need to persuade others: a possible indication of the students adopting an outward focus and shifting from a need for understanding to a need to convince others. Some evidence of this shift is suggested in the transcript below:

71.	Luna:	A table for measurements. Because if we can come up with three or four pyramids that have a scalene side we could measure them and show they're not all the same.
72.	T:	Why would you want three or four?
73.	Luna:	To show that there is not only the one.
74.	T:	So if there is three or four, and you can show that there are three or four, what benefit do you see to having three or four rather than just one?
75.	Luna:	To convince
76.	T:	Wouldn't one convince me?
77.	Laverna:	Not always.
78.	Luna:	Maybe
79.	T:	Maybe. What would be the advantage then of having three or four?
80.	Luna:	It would convince you more.

81. Sally:		We are going to show the completed model of a scalene pyramid and show a net and in front of the audience we are going to put the net together to show a scalene faced pyramid.
82. T:		Why are you going to do it in front of the class?
83. Sally:		So they can believe the net does make a scalene sided pyramid.

These discussions indicated that students were experiencing little problem envisaging the evidence that might be required to demonstrate a scalene-sided pyramid. At this stage the mathematical context did not create difficulties; although the students did need to repeatedly refer to their workbooks, maths dictionaries or the internet to remind themselves of what the properties of scalene triangles and pyramids were. Furthermore, the absence of context removed an additional level of complexity. Thus the students could apply their geometric knowledge to consider the nature of evidence more deeply. In this respect their familiarity with the mathematics served to scaffold the students developing appreciation for the epistemic basis of the evidence.

Persuasion was considered to be the level of argument at which the students would be most open to being challenged (Berland & Reiser, 2009) and when the hypothesised (Figure 4.4) reliance on evidence and evidence quality would strengthen. In the excerpt above, we can see that this is the case with the students in this research group. Their focus, rather than being on the use of persuasive devices, was in obtaining further evidence to strengthen their claim; although, it is unclear whether students were considering the possibility of being challenged, or trying to reinforce their case.

Once the students had the opportunity to develop an idea of the evidence they wanted to focus on, the class was brought back together to share. One significant advantage of this collegiate sharing was that students were able to question each other's ideas and thus refine and improve their own plans. Each group determined that the evidence they would present would consist of some (or all) of model, diagram and net, with measurements marked to 'prove' the scalene nature of the scalene face. Another advantage of the collegiate sharing was that in instances where the group was struggling to move forward, the presentation of ideas from others acted as a catalyst for their own ideas. In this instance the students were unable to determine a solution to one particular quandary: what evidence would represent the converse claim? In theory, to confirm that a scalene pyramid is possible, only one accurate example would need to be provided; but this is not true of the converse "*A pyramid cannot have a scalene face*". Discussion as a whole class at the completion of the group presentations enabled the students to consider this difficulty:

84.	Connor:	For the evidence that [the scalene pyramid] doesn't work, you would need to show a big range of different nets that didn't work and we'd have to show each measurement that you tried that didn't work, each face your tried that didn't work, and each net that didn't work.
85.	T:	So it sounds like it would be a lot harder to prove that you can't than it would be that you can.
86.	Oliver:	Yes, 'cause you would have to have all the measurements, like heaps of different measurements, and you would have to have heaps of different faces, and then you would have to have...
87.	Connor:	Yeah - you would have to have more than one net that didn't work. If you just have one it would show that you didn't pick a good shape to start with or the shape didn't work so it doesn't really prove anything. It is easier to prove that it works because that way you only have to show one net that works, you don't have to show a range that works.

In the Barbie unit, we saw that students initially struggled to envisage the kind of evidence that would be helpful. In this unit, they demonstrated a better understanding of the role evidence plays, and a sense that they knew the mathematics required which enabled them to launch into considering possibilities for evidence immediately and this led quickly to students wanting to obtain their evidence. However, a focus on quality evidence, in terms of epistemic criteria, was in this instance conceptualised more easily than achieved. Some of the issues the students came up against challenged their geometric knowledge and caused them to struggle more than they, or the teacher had anticipated.

7.3 Epistemic Criteria for Evidence

In the previous unit (Section 6.4), the basis for identifying specific epistemic criteria for acceptability and quality of evidence, and for evidence-getting was discussed. These criteria addressed the epistemic acceptability of students' arguments:

- Gathering of evidence,
- Organisation and representation of evidence, and
- Interpretation and analysis of evidence.

The first two criteria will be addressed conjointly as the data were gathered through the construction of models and diagrams; that is, through evidence representations. The students commenced data gathering by uniformly seeking to construct their planned pyramids. While the students could easily envisage the evidence they needed, the teacher recognised that they did not yet have the mathematical knowledge required to go about obtaining that data, and nor was it appropriate to introduce the level of mathematical content required to enable to students to consider this problem in an abstract manner. This was to be a source of conceptual challenge in this inquiry.

7.3.1 Gathering, organising and representing the evidence

In the curriculum, students typically spend some time prior to Year 5 in constructing three dimensional geometric shapes from provided nets or construction equipment designed for the purpose (for example, Geoshapes): or in deconstructing existing shapes, such as cardboard boxes, to determine the net. In either case, these fit together without difficulty and almost always form the required shape quickly and easily. This may have given the students a false sense of simplicity. In this instance, the approaches taken varied but the students consistently struggled far more than they had anticipated: largely because most began by drawing a pyramid net (or what they conceived a pyramid net to look like) and then trying to make it 'fit'. This proved to be extremely difficult as essentially the students' only real way forward was to be to attempt to build their intended models (or nets) through trial and error. This afforded the opportunity for the students to struggle which in turn enabled deeper engagement and a stronger understanding and appreciation of the pyramids and their attributes. While students initially had small difficulties with issues such as the shape of the faces (a few triangular prisms were initially designed) and the number of faces (which would depend on the shape of the base), these were self-adjusting errors: the students could identify them quickly and easily themselves. As the students had been engaged in previous study of pyramids, they were all aware that pyramids could have any polygon as a base; however, most commenced with square-based pyramids before shifting to experiment with other base shapes. Two issues which caused greater difficulty were the lengths of the sides and the internal angles of the faces.

Length of sides

One of the first attributes the students struggled with was the issue of ensuring that the sides that would form the pyramid edges were of the same length. The groups who started by drawing a net and then folding it were having particular difficulty with this idea.

88.	Kody:	Well I tried to make a pyramid but it doesn't really work out.
89.	T:	What do you notice about the adjacent sides here [indicating]
90.	Zachary:	That's too long (indicating one of the adjacent sides)
91.	T:	So for those sides to be able to touch each other, what are you going to need to do?

A significant breakthrough came when Delmar realised that adjacent sides (on the net) had to be of equal length. As more of the students struggled with this problem, the teacher asked Delmar, who had worked it out himself, to share what he had discovered as part of a class discussion. Delmar drew a diagram on the board and indicated the edges as he talked. His diagram is replicated in Figure 7.4.

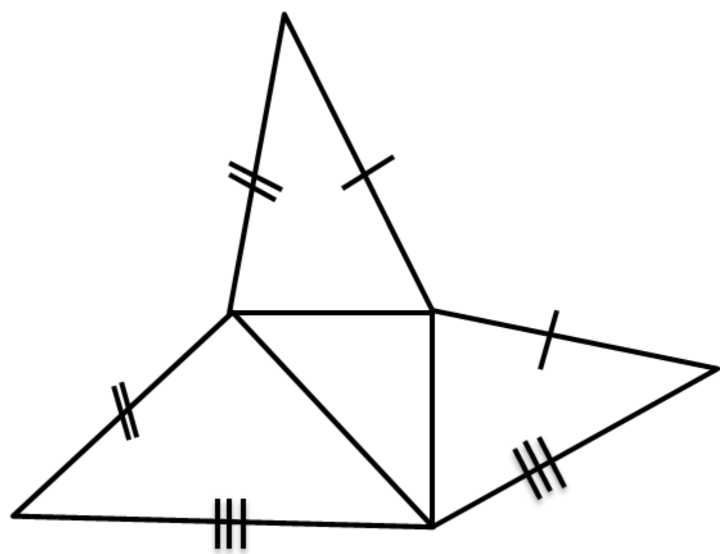


Figure 7.4: Labelling the net to indicate equal lengths

92.	Delmar:	So you have to make those exactly the same (points to two adjacent triangle sides which will come together to form a common edge) otherwise one side of the face won't reach up to the other.
93.	T:	So show me which ones have to be the same as which other ones? [Delmar pauses]
94.	T:	Do you remember how to mark something that is the same length?
95.	Delmar:	Oh yeah. So you have to make that and that the same [indicates adjacent sides in the net] because when you join them up [folds his hands to indicate meeting]. You also have to make that and that, and that and that [indicating adjacent sides which will pair to form edges] the same.
96.	T:	Does that have to be the same as that one? [indicates two sides of same triangle]
97.	Delmar:	No.
98.	T:	Tell me something. Is this a general rule? I mean if I build any pyramid now that follows this rule, is it going to work? [draws a net in which every pair of adjoining sides are equal length]. Is that always going to make a pyramid?
99.	Delmar:	It should.

This interchange initiated an “improvable theory” approach which hadn’t actually been intended but which characterised the higher level of Knowledge Building as discussed by Bereiter and Scardamalia (1996) (refer to Section 2.4.2). Here Delmar has provided a hypothesis that, so long as the adjacent edges were of the same length, a pyramid should be able to be formed. After this discussion, the students returned to the task and spent some time engaged with building pyramids: assisting and guiding each other as necessary. However, they quickly discovered that even when a net was constructed that met the criteria for adjacent edges being the same length; they still failed to form pyramids. It was at this point that the students began to experience difficulty and the teacher instigated a class discussion to draw out what else needed to be considered.

Size of angles

100.	T:	With every triangle there are two things that change. What are the two things that change with every side on your triangle? [wait time]
101.	T:	One is the length...and the other one is the?
102.	Lucy:	The degrees, angles.
103.	T:	The angles. So those are the two things that you need to think about when you are making your triangle. So for every side you have to get both the length right and the angles right. Now it is not the lengths that are hard. When you have a look at your net, it is very easy to say 'well this is 8cm so this one has to be 8cm and this one is 12cm so this one has to be 12cm'; it is working out what these angles need to be.

The level of mathematics the students required to address this problem was considered well beyond their level of development and there was limited opportunity for simplifying or scaffolding such knowledge. Rather than 'giving' the students the answer, the teacher invited students who had found any method of building a pyramid to share their approaches and explanations, in accordance with a Knowledge Building focus.

Alternative approaches to the problem

Three different, successful, approaches adopted by the students were shared with the class and are illustrated in the students' own words below.

104.	Geneva:	I found that to make sure they are the right, that the sides are the right shape, if you have like a certain plan in your mind, like having two scalene faces on a square-based pyramid, then um, but you don't know what the sizes of the other two face have to be, then you can just make um, a big amount of space and then you can use your ruler and the rest of the shape and umm find out what the faces, what the shapes of the faces are supposed to be. [modelling visually]

105. Sally: Well I drew a scalene triangle and I cut it out then I traced around it to make like one of these [a face] and so I cut it out and traced around it and kept going.

106. Lucy: I thought maybe you could get a pyramid with equilateral sides and then just adjust them to make it scalene and then see if that would work.

These examples, provided by classmates, enabled those students that were struggling to quickly find a way forward. At the conclusion of this discussion, all students were able to use one of these methods, or a method of their own inspired by these methods, to construct a scalene pyramid.

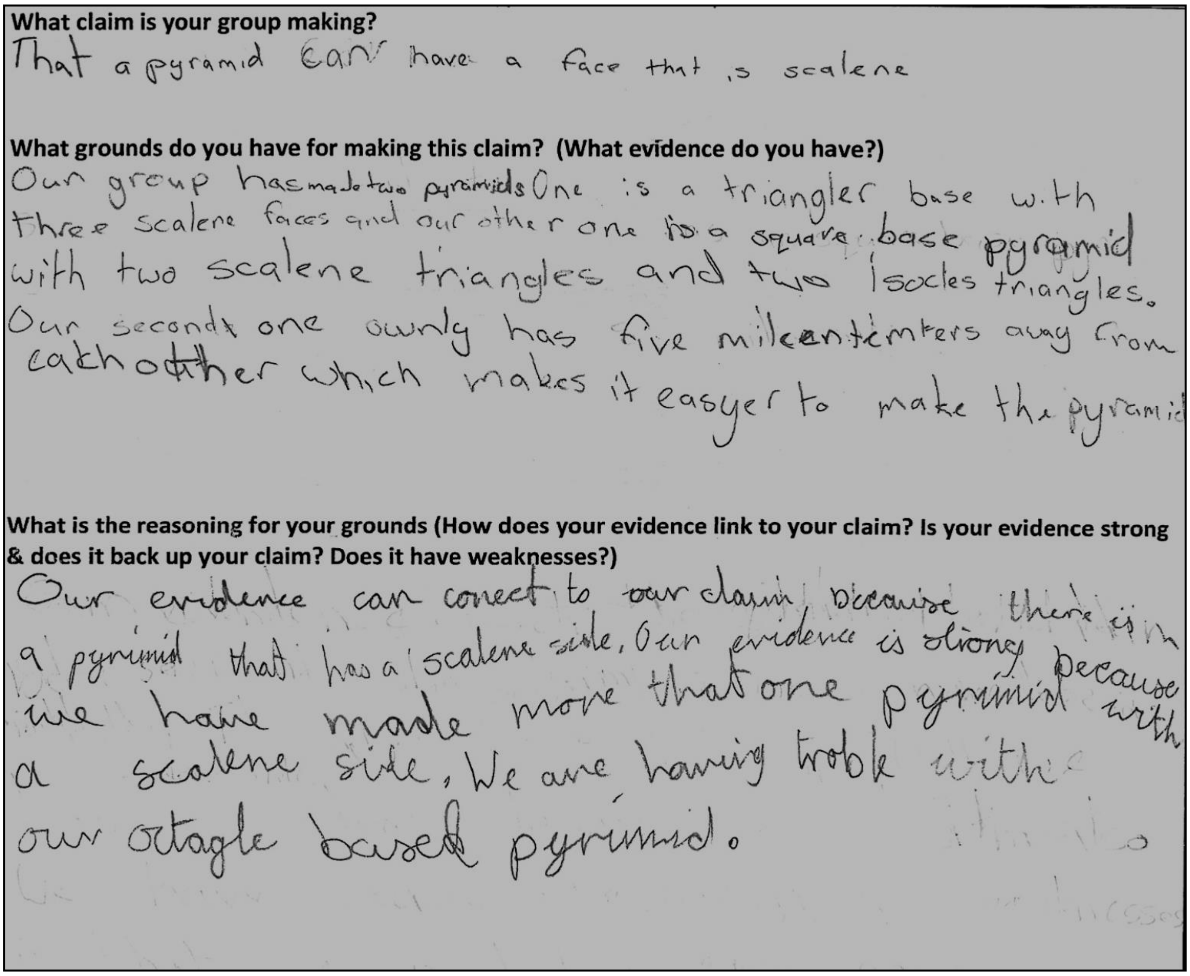
7.3.2 Interpretation and analysis of the evidence

In the first unit undertaken (Barbie), the students had been required to apply informal statistical reasoning to interpret and analyse data. In this instance, the interpretation and analysis took a different form. First, the focus was on continuous improvement: the students did not gather the data and then make a decision from the data. Rather they assessed the quality of their models and nets continuously as they were constructing them in order to determine the acceptability of their construction. Second, as students were receiving this immediate feedback from the task itself (they could see if the design was not working), they were able to use ongoing analysis to adjust their design. Thus the success of the students' interpretation and analysis could be measured from the models and nets that they presented in connection with the claim they made. Would they claim to have built a scalene-faced pyramid which was represented by something clearly not scalene, or not a pyramid? Would they acknowledge limitations? In order to assess the students' geometric reasoning, they were tasked with completing a scaffolded written argument which was then assessed using the criteria developed previously (Argument Structure and Epistemic References, Section 5.7.2).

7.3.3 Framing the written argument

The importance of the role of audience to argumentation was addressed briefly in the theoretical framing of this thesis (see Section 3.6.4) and implicitly suggests that students might have more difficulty in developing a written argument than in producing an oral one because of the lack of obvious presence of audience. In delivering an oral argument, any overlooked points or areas requiring clarification can be queried or addressed by the audience, enabling the proponent of the argument the opportunity to respond in a more natural way. Whereas a written argument takes the form of rhetoric, where the audience is implied by a potential reader; however, this reader may be thought of by the student as both physically and temporally distant, if thought of at all. However, there are many instances where a written argument is required; for example, a project proposal or an engineering solution. Even when a written argument is not required, studying the components of such enables a stronger, more coherent argument to be developed. Thus it was deemed important that students learn to construct and identify the component parts of the argument in a written form.

The students, in their groups, had now all constructed at least one scalene-faced pyramid and/or the net for such. To establish an indication of the students' early attempts to construct a written argument, the students were provided with a scaffolded worksheet and requested to complete the sheet in preparation for presenting their argument to the class. Students were requested to respond to claim, evidence and reasoning with prompts. An example is provided in Figure 7.5 with the corrected text included below for ease of reading. The students were requested to work individually to complete the sheet in order to determine whether all students in the group had understood the process and concepts.


What claim is your group making?
That a pyramid can have a face that is scalene

What grounds do you have for making this claim? (What evidence do you have?)
Our group has made two pyramids One is a triangler base with three scalene faces and our other one is a square base pyramid with two scalene triangles and two Isocles triangles. Our second one ownly has five mileantemters away from eachother which makes it easyer to make the pyramid

What is the reasoning for your grounds (How does your evidence link to your claim? Is your evidence strong & does it back up your claim? Does it have weaknesses?)
Our evidence can conect to our claim because there is a pyrimid that has a scalene side, Our evidence is strong because we have made more that one pyrimid with a scalene side, We are having troble with our octagle based pyrimid.

What claim is your group making?
That a pyramid can have a face that is scalene.

What grounds do you have for making this claim?
Our group has made two pyramids. One is a triangular base with three scalene faces and our other one is a square base pyramid with two scalene triangles and two isosceles triangles. Our second one only has [side lengths] five millimetres away from each other which makes it easier to make the pyramid.

What is the reasoning for your grounds?
Our evidence can connect to our claim because there is a pyramid that has a scalene side. Our evidence is strong because we have made more than one pyramid with a scalene side. We are having trouble with our octagonal based pyramid.

Figure 7.5: Example of Sadie's initial scaffolded argument (spelling corrected)

In order to monitor students' progress in the early stages of the unit, an overall assessment of these arguments (Table 7.1) was made against the criteria sheet used in the Barbie unit and found in Section 5.7.2 in full.

7.4 **Assessing the Student's Initial Arguments**

The student's initial, scaffolded arguments in this unit (example provide at Figure 7.4) were once again assessed against the criteria sheet developed for the purpose (overall responses summary is provided in Table 7.1). A brief discussion of the salient scores is provided in this section, organised into Argument Structure and Epistemic References.

7.4.1 Argument structure

In the previous unit, the students had been provided with the claim "Barbie does have human proportions" or "Barbie does not have human proportions". Thus they had little experience of developing their own claims. This is often considered a difficult aspect of constructing an argument (Zembal-Saul et al., 2013) and it is important that the students learn to do so clearly and succinctly, and in such a manner as aligns with their evidence (Sampson & Clarke, 2006). In this instance, all students, with one exception, clearly and succinctly made a claim statement. The other claim lacked only in clarity: '*It can have a scalene side'* (Item 3, Table 7.1). As qualifiers had not been specifically addressed, it was unsurprising that there was limited use of such (Item 7, Table 7.1).

The evidence presented by the students was promising, comprising references to nets and models of scalene-faced pyramids (Item 4, Table 7.1). The students had not required support from the teacher in order to identify evidence they felt would assist them to address the question and make a claim. In the previous Barbie unit, the students were heavily supported and scaffolded prior to making their first claim, and the evidence had been collectively gathered. Thus the students' challenge was greater in the Pyramid unit as they were afforded increased autonomy.

In terms of the reasoning provided (Item 5, Table 7.1), students were able to describe evidence which was relevant; however, the detail was insufficient to enable the claim to be convincingly supported. Likewise, while the reasoning in most instances reflected the evidence to claim link, there was a lack of specific information; for example, *"Our evidence links to our claim because it proves it is scalene".* The lack of detail in both the evidence and the reasoning impacted on the quality of the epistemic references.

Table 7.1: Assessment of students' first arguments (mean Likert scale score: range 1 (low) - 5 (high))

	Indicator	Descriptor Within Community Standards:	Initial Response (n=20)
	Argument Structure		
1	Research Question	The research question is clearly and specifically stated	a*
2	Research Question - Context*	The research question informs the wider research context	b*
3	Claim	The claim is explicit, foregrounded and references the question	4.9
4	Evidence (Grounds)	Evidence reflects audience, is relevant to the research question, contains sufficient (but not extraneous) detail	3.2
5	Reasoning	Reasoning co-ordinates logically and considers all evidence	3.3
6	Claim – Context*	Claim implications for wider context are explicit	b*
7	Qualification	Qualifier is provided with details as to when it is applicable	1.2
	Epistemic Reference		
8	Evidence Collection	Evidence collected / generated responds to the question being asked	3.0
9	Foundation for the Evidence	Evidence provided is data-based (as distinct from fallacy, conjecture, opinion)	4.3
10	Evidence Gathering	Methodology for obtaining evidence is provided and is appropriate	c*
11	Evidence Organisation / Representation	Representation/ organisation of data are accurate and appropriate for the audience and purpose	2.1
12	Evidence Interpretation / Analysis	Interpretation / Analysis of evidence meets community expectations: accuracy, clarity, method, efficiency	3.1
13	Evidence Anomalies or Contradictions	Any anomalous or contradictory evidence is provided and addressed factually or in terms of limitations	c*
14	Reasoning	The justification for making a claim, based on the evidence, is suitable given the community of mathematical learners	2.4

a* - information that was provided by the teacher and therefore not assessed
b* - no wider context was applicable to this question
c* - the nature of this inquiry was such that these were not relevant to the inquiry

7.4.2 Use of epistemic criteria

In the early stages of the Barbie unit, initial responses to the question asked were intuitive. In this unit, a shift away from intuitiveness was noted with students instead spending time considering the evidence they could ideally present to make and support their claim. In all instances, this evidence was solely, or almost solely, based on mathematical underpinnings, as distinct from supposition, conjecture or opinion (Item 9, Table 7.1). One of the most significant difficulties for the students was in representing or organising their evidence (Item 11, Table 7.1). In most cases, students elected to either construct a model of a pyramid, draw a net diagram or both. While this in theory was a logical approach, in practice, the students had difficulty with the accuracy and the technical skills expected of a geometric task. For instance, many of the pyramids had been forced into position, which caused faces to deform, or prevented edges and corners from meeting properly. In turn, the questionable nature of the representations was not addressed in the analysis and interpretation of the phase (Item 12, Table 7.1), and this cast doubt on the strength and validity of the claim.

In expressing their reasoning, there was an expectation that students would demonstrate a clear link between the evidence collected and the claim made (Item 14, Table 7.1). In most instances, the students were able to provide the link between the pyramid representation (or their description of a representation) and the claim they had made, for example: "*Our evidence links to our claim because we have done a scalene pyramid*". However, ideally, the students were also to provide evidence that the pyramid was scalene (that is, had at least one scalene face) and was in fact a pyramid (as one group erroneously produced a scalene-based triangular prism), by presenting the known geometric attributes of these shapes.

7.4.3 Comparison with previous unit

Shifts from the students' positions in the Barbie unit were observable, particularly in the carry-over of concepts from their previous argumentation unit, undertaken more than 6 months previously (a comparison is provided in Table 7.2). In particular, students had no difficulty in constructing their claims explicitly. The students had moved from a position of providing *opinion or conjecture* as their first supporting grounds for an argument, to immediately considering what *evidence* might be required in order to support their argument, and ensuring it was mathematically- based. As the units have changed context from having an externally contextualised focus (using mathematics to solve a problem that is not contextualised in mathematics) to solving a problem contextualised squarely within the mathematical domain, it is difficult to establish how much of this shift should be ascribed to the repeated exposure of ideas and how much to the shift in nature of context.

The first construction of an argument in the Barbie unit suggested little difference in Epistemic Reference scores from the first construction in the Pyramid Unit (Table 7.2). This was not surprising as the mathematics applied in the Barbie unit was associated with proportional and statistical reasoning, and the Pyramid unit was seated within the field of geometric reasoning and spatial awareness. As the strands of mathematics were different, it would not be supposed that development of mathematical knowledge in one would necessarily lead to development in the other. Ideally, to identify shifts in learning, we need to look across units for development of Argument Structure, and within units for changes in Epistemic Reference, as content is more heavily reflected in Epistemic Reference.

Table 7.2: Comparison in evaluation of novice argument in first (Barbie) and second (Pyramid) units (mean Likert scale score: range 1 (low) - 5 (high))

	Indicator	Descriptor Within Community Standards:	"Barbie" Initial Response n=25	"Pyramid" Initial Response n=20
	Argument Structure			
1	Research Question	The research question is clearly and specifically stated	a*	a*
2	Research Question - Context*	The research question informs the wider research context	a*	b*
3	Claim	The claim is explicit, foregrounded and references the question	a*	4.9
4	Evidence (Grounds)	Evidence reflects audience, is relevant to the research question, contains sufficient (but not extraneous) detail	3.5 (c*)	3.2
5	Reasoning	Reasoning co-ordinates logically and considers all evidence	2.9	3.3
6	Claim – Context*	Claim implications for wider context are explicit	a*	b*
7	Qualification	Qualifier is provided with details as to when it is applicable	1.0	1.2
	Epistemic Reference			
8	Evidence Collection	Evidence collected / generated responds to the question being asked	a*	3.0
9	Foundation for the Evidence	Evidence provided is data-based (as distinct from fallacy, conjecture, opinion)	a*	4.3
10	Evidence Gathering	Methodology for obtaining evidence is provided and is appropriate	a*	d*
11	Evidence Organisation / Representation	Representation/ organisation of data are accurate and appropriate for the audience and purpose	1.9	2.1
12	Evidence Interpretation / Analysis	Interpretation / Analysis of evidence meets community expectations: accuracy, clarity, method, efficiency	3.4	3.1
13	Evidence Anomalies or Contradictions	Any anomalous or contradictory evidence is provided and addressed factually or in terms of limitations	3.0	d*
14	Reasoning	The justification for making a claim, based on the evidence, is suitable given the community of mathematical learners	2.4	2.4

a* - information that was provided by the teacher and therefore not assessed
b* - no wider context was applicable to this question
c* - significant guidance through in-class discussion and collaborative planning
d* - the nature of this inquiry was such that these were not relevant to the inquiry

7.5 Argument as a Process – Communicating the Argument

Engaging students in the highest level of argumentation – persuasion – was hypothesised to promote a reliance on evidence and increased evidence quality (Figure 4.4). Thus students were provided with the opportunity to communicate their arguments to the class to: a) create a necessity to formulate ideas in an articulate manner; b) enable students to identify the claim, evidence and reasoning in each other's arguments and to challenge and provide feedback on these components; and, c) enable students to consider the strength of the epistemic references being used and to provide feedback or challenge to those references. In this way, both the presenting students and the audience were developing their existing knowledge, while at the same time assisting others to strengthen their arguments (by improving their epistemic and thus mathematical understanding). As the students were quite adept at both articulating their stance and identifying claim, evidence and reasoning components, it was this last purpose, considering the strength of epistemic references, which was particularly worth noting in the unit.

7.5.1 Engaging in persuasion

It was apparent in the Barbie unit that, when the students were given the opportunity to question their classmates' arguments, they initially found doing so quite challenging. As a consequence, the teacher had needed to model the questions for the students before the students ultimately mimicked those questions to ask as their own. In this Pyramid unit, a very different scenario was noted. The students immediately engaged with each other's presentations. The interactions the students engaged in were open-coded to identify interaction 'types', with three distinct categories being apparent: clarifying questions; positive/constructive feedback; or comments/questions which served to challenge the student to consider or to extend their argument/reasoning. By way of Illustration, a presentation transcript has been provided below with the questions asked by the class in the dialogue following.

Group presentation

107. Jeff: Our claim is you can have a pyramid with a scalene face.

108. Zachary: Our evidence is that we have one here with a scalene face. Here are the measurements to show that we have a scalene face. All of the measurements will be different because a scalene triangle has all different measurements

on each edge.

109. Kody: This edge is 9.75 cm, this edge is 8.6 cm and this edge is 12.5 cm. Now we are going to Samuel so he can tell you our reasoning.

110. Samuel: You can make a scalene faced pyramid but you have to have a rectangular base. I don't know, neither does Zachary or Kody, if you can use any other base that has the ability to consider [sic - construct?] a scalene face as well.

Below are the questions asked by the class, with the category of question identified in parenthesis at the conclusion of each question. The entire dialogue has not been included in the questioning, merely the first comment/question that led to each discussion. As can be seen, the first five of the six discussions that originated from this presentation were student initiated.

Questions addressed to the group above

111. Lucy: Did you try any other based pyramids?... [clarification]

112. Shana: I couldn't really see the pyramid, could I just see it, and the base… [feedback]

113. Dominica: In your reasoning that was really helpful how you said that none of you could see that you could find any other base. You might just say "we didn't find any base" but it’s a bit more powerful saying 'none of us'. … [feedback]

114. Leticia: When you measured the scalene pyramid, we don't know for sure if it was actually... the measurement because we couldn't actually see….[feedback]

115. Connor: I liked the qualifier in the reasoning because um um um. The part where they said that you can make it but they said that they only found out that you can use the rectangular-based…. [feedback]

116. T: My question is you have shown that one of those sides is scalene, what have they not shown me? [that it is a pyramid]… [challenge]

Equally noteworthy was the nature of the comments/questions asked. The table below (Table 7.3) provides a comparison of the number and nature of teacher's questions and those of the students after each group presentation. The majority of the questions were posed by the students: many of which served to address the quality of the evidence and were often addressed from the position of the student not being 'convinced'. By contrast, few questions came from the teacher and of those that did, nearly half were of a clarifying nature. The indicators in the table are again organised according to the Epistemic Reference indicators devised for evaluating arguments in this study (Section 5.7.2).

Table 7.3: Comparison of number and type of questions asked by teacher (T) and by students (S) during presentation of group arguments

Indicator	Descriptor Within Community Standards:	T	S
Epistemic Reference			
Evidence Collection	Evidence collected / generated responds to the question being asked	1	3
Foundation for the Evidence	Evidence provided is data-based (as distinct from fallacy, conjecture, opinion)	-	-
Evidence Gathering	Methodology for obtaining evidence is provided and is appropriate	-	-
Evidence Organisation / Representation	Representation/ organisation of data are accurate and appropriate for the audience and purpose	-	7
Evidence Interpretation / Analysis	Interpretation / Analysis of evidence meets community expectations: accuracy, clarity, method, efficiency	2	3
Evidence Anomalies or Contradictions	Any anomalous or contradictory evidence is provided and addressed factually or in terms of limitations	-	2
Reasoning	The justification for making a claim, based on the evidence, is suitable given the community of mathematical learners	1	2
Other		-	-
Clarification of details		3	3

The issue most commonly raised was related to the representations, in this case, typically models. It was noted in the students' individual arguments that the quality of the models was somewhat lacking; in particular a certain amount of 'encouragement' had been applied to ensure the models fit together. The students determined that they could check the accuracy of each triangular face by totalling the internal angles of each triangular face, and also that they could establish whether a triangle was scalene by the length of its sides. Once they had done this, they were keen to challenge triangles that were potentially isosceles rather than scalene. The students became quite critical of each other’s evidence and uncovered several instances of triangles that were purported to be scalene; yet, a small difference in the length of two sides with a corresponding anomaly in the angles suggested they were more likely isosceles. This led to the students expecting a higher standard for acceptability and they likewise promoted increased accountability for their own evidence.

7.5.2 Use of qualifiers

A further issue that arose during these class discussions was the use of qualifiers. When working with ill-structured problems, it is often necessary to refine the parameters of the questions and then define or explain what decisions were made. In Science, these decisions may be built into the methodology and explained in terms of assumptions or limitations to the research design, or as parameters of the research. In mathematics, as in science, solutions need to be able to be replicated or verified, and an absence of articulation of underlying assumptions limits that ability. If we postulate a claim we need also to clarify under what conditions that claim holds true. In argumentation, this is often the role of the qualifier. At the outset of this unit, the researcher considered that qualifiers, while a formalised part of Toulmin’s framework (Toulmin, 1958), were likely too complex for such young students. However, it was early in this unit, during a whole class discussion, that one group offered a segue into production of a qualified argument that the teacher chose not to overlook:

117.	T:	You said you are going to try to make three different types of pyramid. Why?
118.	Lucy:	To see if only the square based pyramid works and not the triangular based pyramid. If you only do the square based pyramid we won't find out that the triangular based pyramid wouldn't work which means you could say "Can a pyramid have a scalene face?" "Yes, if it is a square-based pyramid,

or a whatever - another one, but not if it is a triangular base" or something.

119.	Shana:	Other people might have just one base [discussion garbled] so just make sure that other bases can work too.
120.	T:	You did sort of answer the question I have asked but you haven't completely answered the question I have asked. You said you want to find out if you can do a triangular-base, square-base, pentagonal-base because of those 'if' type questions, "if you can do this but can't do this?" but what are you actually trying to do in terms of your evidence here?. By saying you want to do the 'ifs' - this might work and this might not, what are you trying to do?
121.	Lucy:	We are trying to get more evidence and make the evidence stronger.
122.	T:	But **are** you making the evidence stronger? **If** you find that you can only do it with a triangular-base, that you can't do it with a square or a pentagonal base, then are you making the evidence stronger? [speaker emphasises words in bold text.]
123.	S's:	No.
124.	T:	You are actually making the evidence weaker but you are also making it more...
125.	Shana:	True.
126.	T:	So you are making it more complete?
127.	S's:	[Nodding in agreement.]
128.	T:	So you would actually end up with a modified claim. We have said that your answer could be 'yes - you could have a pyramid with a scalene face', 'no- you can't have a pyramid with a scalene face'. They're [the group] are actually saying there might be a third answer. There might be a 'yes you can if it has such and such'. So they're actually modifying the claim and then trying to find evidence of a modified claim.
129.	Shana:	Are we allowed to?

130. T: Of course! But there is a special name we give to this type of adjustment; it is a qualifier.

Toulmin's Model (1958) identifies the role of a qualifier specifically. Thus, if the student argument was to be formally deconstructed into a Toulmin model, it would appear as the argument in Figure 7.6 (with warrant, rules and backing taking the function of reasoning and the grounds being 'evidence'). The CER model (McNeill & Martin, 2011) however, makes no provision for the use of qualifiers. This was managed by introducing the idea of a qualifier as an optional appendage to a claim: "A pyramid can have a scalene face *if it has a square or triangular base*". Due to the students' need for a qualifier, it was later incorporated into the student CER model. An example has been provided later in this chapter (see Figure 7.7).

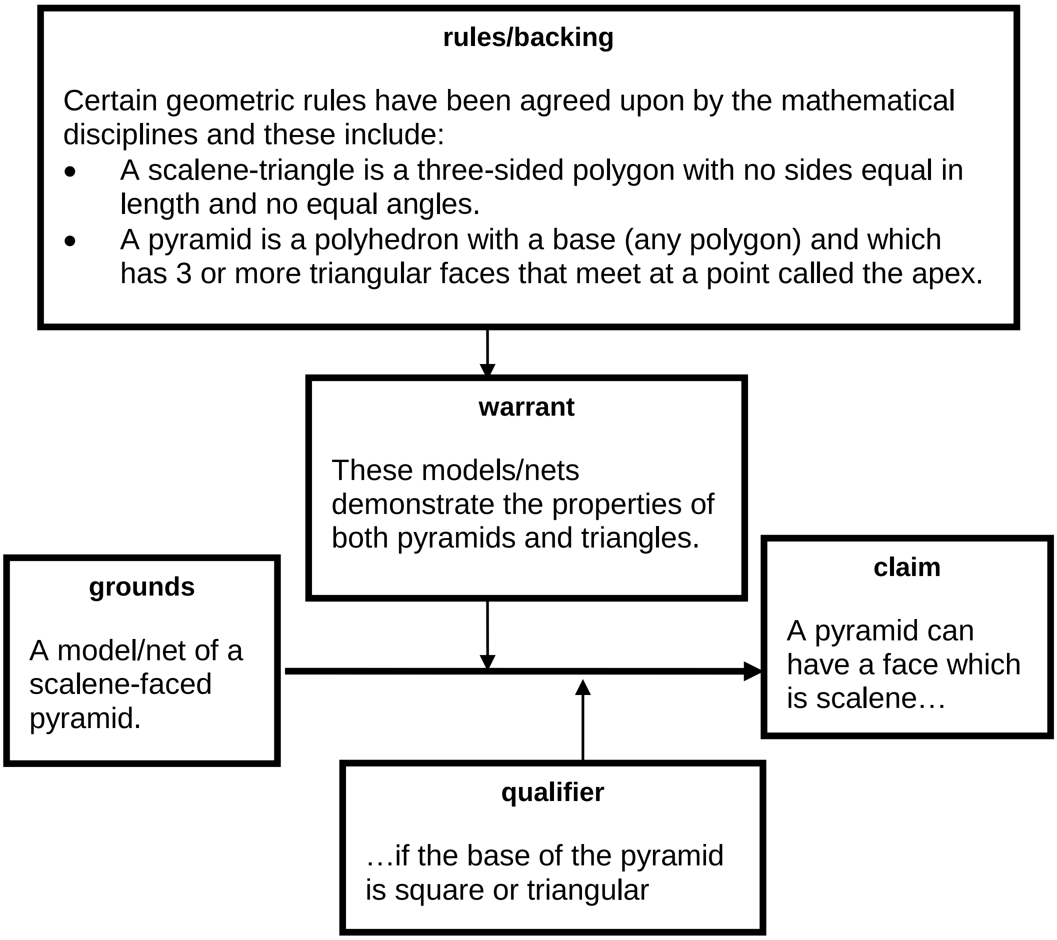


Figure 7.6: The student argument deconstructed into a Toulmin's framework

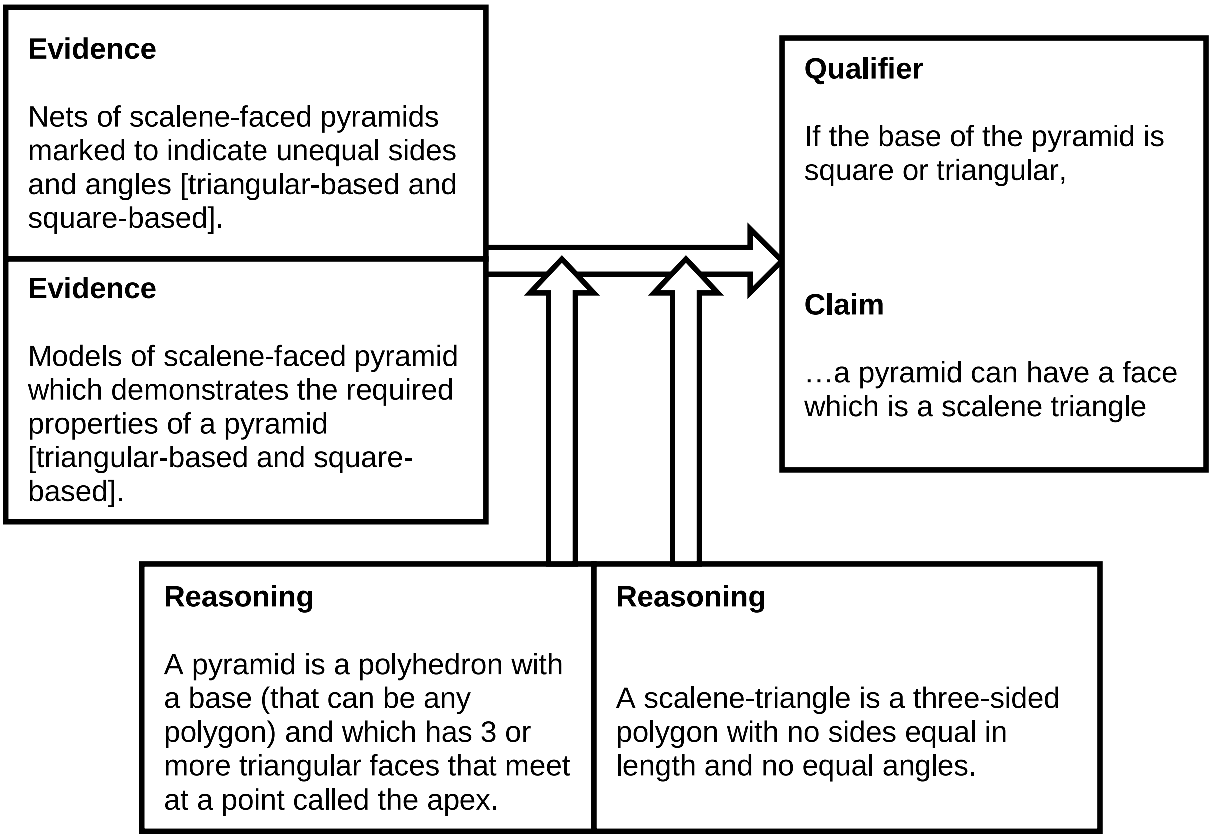


Figure 7.7: A CER representation of the student argument presented in Figure 7.6.

7.6 Argumentation Framework

At the completion of the student presentations, the class engaged in a discussion on the overall argument model and began to work with the teacher to adjust and refine the model for class reference. What started as a simple discussion became quite involved, with the students extending the discussion into the nature of evidence in an argument.

7.6.1 Developing a model for student use

The students were well aware of the research taking place and had been co-opted as research partners early on. This message was reinforced by the teacher as she often prompted them to contribute theoretically to the research. For instance, all parts of the Evidence Model on display in the classroom were individually mounted magnetically, and students had whiteboard markers available, so they were able to ‘play’ with different configurations (and terminology) as they chose. They also knew that the model they were working with was being built as a ‘prototype’ for other students and so knew their role was to make it a student-friendly tool. As such, they were accustomed to raising issues and providing feedback both as ‘students’ and ‘researchers’. They knew their feedback was welcomed and they took this role extremely seriously. In this section, the framework used with the students was challenged, initially by Shana and then later as multiple students become involved. It is reported here as, while no significant changes were actually decided upon, the discussion the students engaged in offers insight into their envisaging of the role of evidence. In particular: the nature of 'Evidence' in the Evidence Model as distinct from 'Evidence' in the Argumentation Model.

In the original model used by the students (Figure 7.3), Evidence was being included both as part of the inquiry model (Question - Evidence - Conclusion) and as part of the Argumentation model (Claim – Evidence – Conclusion). However, Shana challenges this and suggests that the diagram should more correctly and efficiently only have evidence once.

131.	T:	Shana explain what you think and why you think we should do this. I think I can see why you are suggesting it.
132.	Shana:	Because there are two evidences. If we move them around we can link in the one evidence to both parts.
133.	T:	OK. So you are trying to work one evidence in there?
134.	Shana	When you put it down there you [tails off]

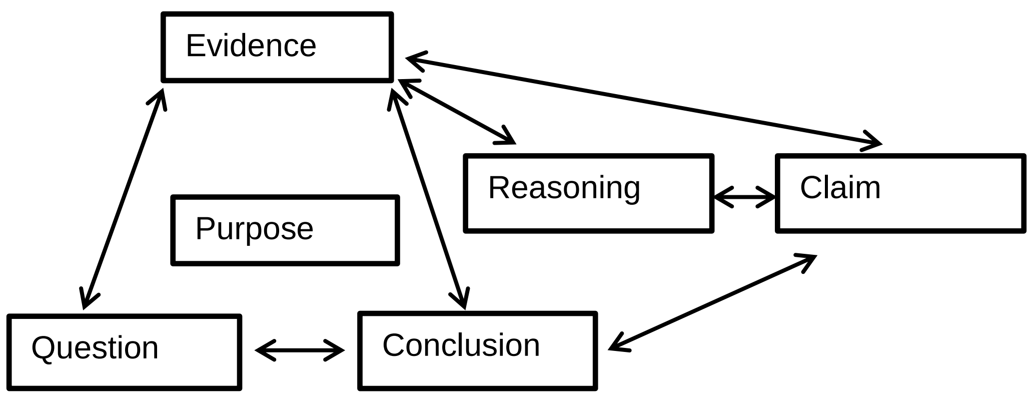


Figure 7.8: Shana's proposed framework

Shana redesigned the diagram to include evidence as one not two labels (Figure 7.8); however, this was challenged by other students.

135.	Connor:	But I know why there are two evidences… There is when the question links to the evidence. That evidence is *finding* the evidence. But then you put your evidence in the conclusion so if you don't put your evidence in the conclusion, then it is just plain reasoning and there is no middle part. You couldn't really make a good conclusion without evidence in it.
136.	Delmar:	Well there is two types of evidences. First evidence is you are trying to find the evidence and the second evidence was something like putting the evidence into the conclusion.
137.	T:	So the first time we approach evidence we are looking at using this evidence to come to a conclusion. The second time we use evidence it is to support our claim. So would the evidence look the same here as it does here?
138.	Delmar:	We kind of analyse it [the evidence] when we want to get to the conclusion.
139.	T:	So this evidence up here (Question-Evidence-Conclusion) is where we analyse our evidence.
140.	Delmar:	Yeah. Because if the evidence is wrong and you finally get to the conclusion, and you haven't checked it. Your evidence doesn't fit your conclusion.
141.	T:	Sometimes, up here we have tonnes and tonnes of evidence. Like you heard around the classroom we had people say, with 68 pyramids that didn't work. But that is still evidence - this kind of evidence here. Stuff that we have found out during the inquiry. Do we need it down here (Claim-Evidence-Reasoning)? Once we have made the claim do we need to keep all our other evidence? Do we need to still say "here are 68 pyramids that didn't work?"
142.	Connor:	No, it is too much.
143.	T:	So sometimes we sift through the evidence and decide what we want and what we don't?
144.	Sts:	Yeah.

The students eventually make the decision to leave the representation as it is in Figure 7.9 as this delineates the different ‘types’ of evidence. However, the students had by now become quite determined to refine the precise nature of the ‘evidences’.

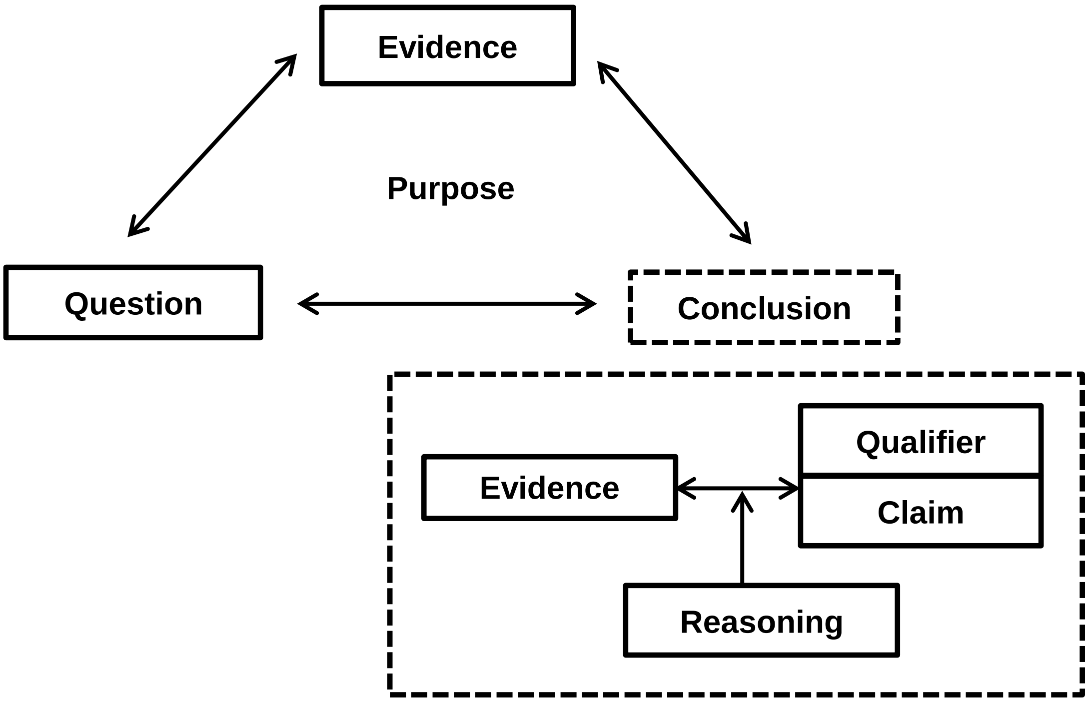


Figure 7.9: Final argument model

In simplistic terms, ‘evidence’ as it is represented in the Evidence Model could be regarded as ‘inquiry evidence’, or the evidence that is gathered, organised and analysed in order to make a claim. Whereas ‘evidence’ in the Argument Model could be regarded as ‘advocative evidence’ where the purpose is to filter and select the relevant evidence that enables a claim to be made and supported. In a Knowledge Building community, it must be stressed that the evidence provided in the argument must seek to provide all the information that is required to enable the community to evaluate the claim: not merely the evidence that supports a claim. In essence, this is where the role of reasoning can be highlighted, as reasoning serves to explain the evidence and the link from evidence to claim.

7.6.2 Developing reasoning

Reasoning is essentially the connector between claim and evidence: it is that which justifies making an evidence-derived claim. In a mathematical inquiry, reasoning essentially explains how the student moved from evidence to claim, and the rules or procedures that enabled them to do so. Figure 7.6 showed the role of reasoning (warrants, rules, backing) if deconstructed into a Toulmin framework. Figure 7.7, showed the same information deconstructed into the CER model as used for this class. In each instance the evidence (a representation of a scalene-sided pyramid), enables the arguer to argue that one can be created. However, what enables the arguer to present this representation as a scalene-sided pyramid? In the field of mathematics there are discipline based agreements (rules) about what constitutes the properties of scalene triangles and pyramids. The reasoning refers to these rules and indicates whether they are met: enabling the model to be held up as evidence. This understanding is essential from the arguer's point-of-view and also from the educators'. It is here that the depth of mathematical understanding resides, and is also the locus of the potential for argumentation to deepen mathematical understanding through the application of reasoning.

As the context of this unit (and the mathematics) was relatively familiar to the students, the teacher took the opportunity to develop the role of reasoning and have students focus on what might constitute reasoning within this context.

145.	T:	Your question is, "Can you build a pyramid with a face which is scalene?". Now you have just explained to me how you could prove that one of the faces is a scalene triangle. What other part of that might you have to prove?
146.	Shana:	That the other faces are scalene.
147.	T:	Not necessarily, the question is whether a face can be scalene - so it wouldn't matter whether it was only one, or more. What else might you need to show? [wait time] So you might have to provide some evidence that your shape is actually a [tails off]
148.	S's:	Pyramid.
149.	Shana:	So we need to say all the stuff about pyramids. So we can see if it has all the factors of a pyramid.
150.	Dominica:	We can also, on the diagram, show the pyramid structure and say what a pyramid would have.
151.	Shana:	And we can mark the angles.
152.	Lucy:	I know why we need to do this. Because even though all the

class knows what it is, if we took it to a different class they might not know what a pyramid is.

153.	T:	You have said here [looking at Lucy's book] 'measurements of triangle'. Why do you want to show me the measurements of a triangle?
154.	Lucy:	I think if I showed the angles [pause] no [pause] the degrees of the angles, it would show it is scalene.

The teacher continued to build the reasoning; developing the understanding that it is not sufficient to simply provide evidence and a claim, that the link between the evidence and the claim must also be specific and must validate the connection (reasoning). Once the students had discussed the importance of reasoning as a group, instances of reasoning, at least in relation to this unit, became apparent in their small group discussions.

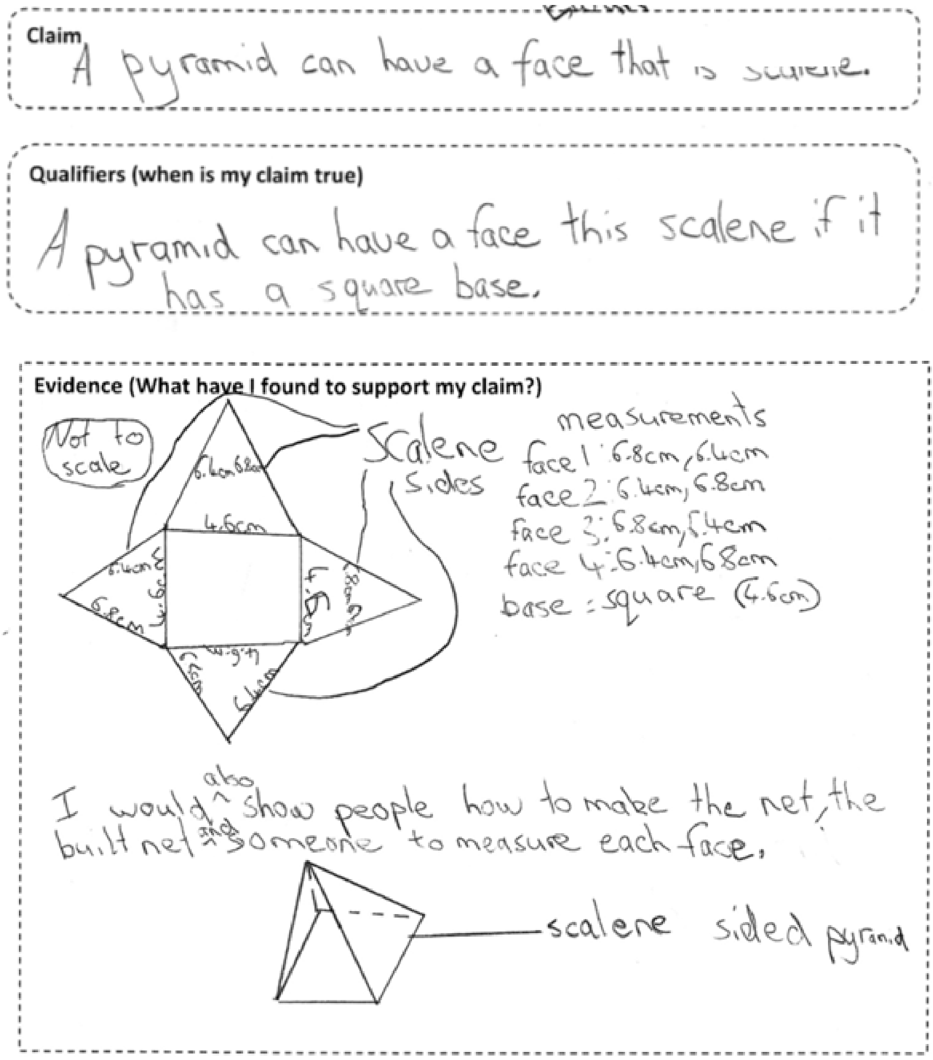
155.	Kody:	…measurements to show that one face was scalene and that the pyramid has an apex.
156.	T:	How do you want to show that the pyramid has a scalene face?
157.	Kody:	A model
158.	T:	And what will the model actually have to convince me that it has a scalene face? Samuel can you think of a way that you could actually show me that it is a scalene face?
159.	Samuel:	Maybe show the lengths [of the sides].
160.	T:	You also said that you wanted to show that it had an apex. Why would you want to show that your pyramid has an apex?
161.	Kody:	Every pyramid has an apex and if it doesn't then it means it’s not a pyramid.
162.	T:	So you're trying to convince me both that it has a scalene face and that it is a pyramid.

Similar conversations continued which introduced many of the students’ uncertainties around pyramids and which had the students hurrying to the internet to determine answers: Is a cone a pyramid? Must the apex be centred over the base? How do you know which face is the base on a triangular-based pyramid? Must a pyramid have a regular base? Can a pyramid's base be any shape? In this way, students developed a richer understanding than they otherwise would have, as few of these issues were addressed in the regular coverage of curriculum content.

After the completion of both group presentations and the class discussion on the nature of evidence, qualifiers, and reasoning, students were given the opportunity to revisit and gather any further evidence they might need in order to develop the best evidence they felt they could; bearing in mind the discussions that had been held in the class. This evidence was to form the basis of their culminating task and enable assessment both in terms of Argument Structure and Epistemic Reference.

7.7 Assessment of Argument Product and Process

This section describes the final task given to the students: a written argument which the students completed individually. The argument served to identify potential student development of argument structure and use of epistemic references in construction of their argument. While content development would appear assessable, content knowledge could be accredited to either actual improvement in content knowledge, or to a student's improved articulation of previously known content. Therefore caution has been exercised in making assumptions about the extent to which the inquiry improved content knowledge. To complete the task, students were provided with a scaffolded worksheet in which they were prompted to provide a claim, qualifier, evidence, and reasoning as a conclusion to the question, "*Can a pyramid have a scalene face*?". A sample response from Connor, selected for the quality of reproducibility and writing legibility, has been included for illustration below (Figures 7.10 and 7.11). Figure 7.10 represents the claim, qualifier and evidence presented by Connor. Along with this, Connor provided an accurate model and net of the pyramid shown, with angles and side lengths marked. While his actual physical pyramid and net diagram were well done, some other students did have trouble creating accurate models due to the manual dexterity required. Despite this, all models approximated scalene-faced pyramids. Figure 7.11 shows the reasoning Connor provided in response to the questions. The extent of the scaffolding of the worksheet can be seen below: students were provided with guided questions to assist with the structuring of their responses. These questions were similarly scaffolded in the first activity (contrast with Figure 7.5), so it is possible to identify components of answers in terms of quality.



Claim

A pyramid can have a face that is scalene.

Qualifiers (when is my claim true)

A pyramid can have a face this scalene if it has a square base.

Evidence (What have I found to support my claim?)

Not to scale

Scalene sides

4.6cm

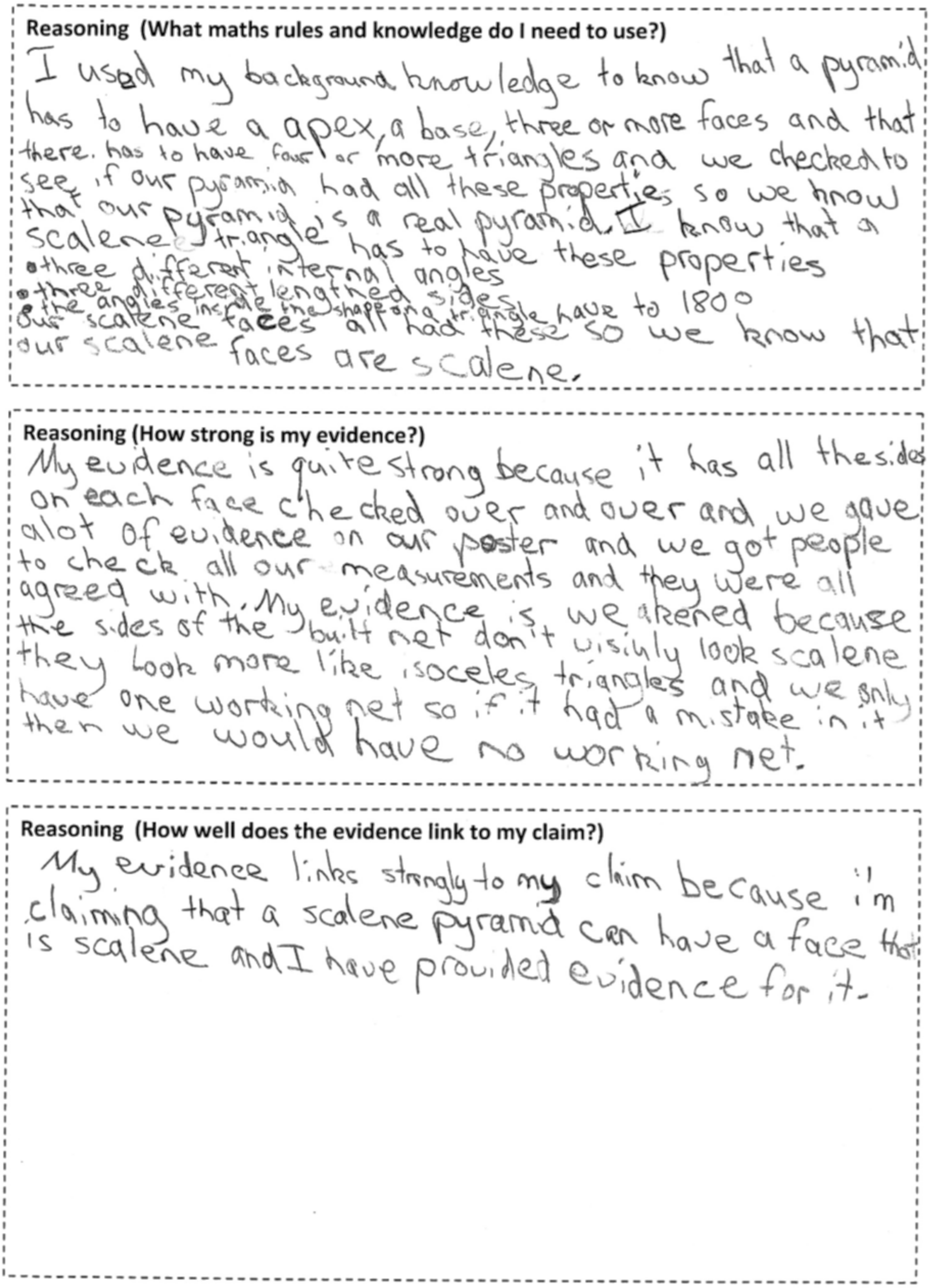
6.4cm 6.8cm

measurements
face 1: 6.8cm, 6.4cm
face 2: 6.4cm, 6.8cm
face 3: 6.8cm, 6.4cm
face 4: 6.4cm, 6.8cm
base : square (4.6cm)

I would also show people how to make the net, the built net and someone to measure each face.

scalene sided pyramid

Figure 7.10 Sample student response to the culminating task – Claim, qualification, and evidence (Connor)



Reasoning (What maths rules and knowledge do I need to use?)

I used my background knowledge to know that a pyramid has to have a apex, a base, three or more faces and that there. has to have four or more triangles and we checked to see if our pyramid had all these properties so we know that our pyramid is a real pyramid. I know that a scalene triangle has to have these properties
- three different internal angles
- three different lengthed sides
- the angles inside the shape of a triangle have to 180

Our scalene faces all had these so we know that our scalene faces are scalene.

Reasoning (How strong is my evidence?)

My evidence is quite strong because it has all the sides on each face checked over and over and we gave alot of evidence on our poster and we got people to check all our measurements and they were all agreed with. My evidence is weakened because the sides of the built net don't visibly look scalene they look more like isoceles triangles and we only have one working net so if it had a mistake in it then we would have no working net.

Reasoning (How well does the evidence link to my claim?)

My evidence links strongly to my claim because i'm claiming that a scalene pyramid can have a face that is scalene and I have provided evidence for it.

Figure 7.11: Sample student response to the culminating task – Reasoning (Connor)

Consideration of aspects of Connor's argument structure shows that Connor was able, as were all students (refer to Table 7.4, presented later in the chapter), to provide an explicit claim; however, on this occasion Connor attempted to qualify his claim by limiting it to pyramids with a square base as this was the only base his group was able to develop a scalene pyramid for. While it is clear what he intended, his wording lacked clarity with the phrase "*this* scalene". In terms of his evidence diagram, the evidence provided is appropriate to the question and contains sufficient detail that it could be replicated by the other students; although they found it easier if the angles were included. Connor did submit a net diagram which was accurate and could be built into a pyramid successfully; however, this too did not display angles. Despite this, it is sufficient to identify a scalene triangle from side lengths alone and the angles on the net were able to be measured if needed. The reasoning applied by Connor indicated that he had the underlying knowledge of the attributes of scalene triangles and pyramids and that these enabled him to identify this shape accurately as a scalene pyramid.

In this example, the epistemic references (Section 5.7.2) employed by Connor demonstrate his responses to be evidence-based and sufficient to address the question. Small errors exist with his representation – the base should have been illustrated as a square for example – although the labelling goes to mitigate that to some extent, and the net provided was more accurate (while this was tested by the teacher it was later 'tested' to destruction by the boys and therefore has not been included here). It was not necessary that Connor account for contradictory evidence as the existence of non-scalene faced pyramids did not preclude or challenge in any way that they could exist. Finally, Connor's reasoning met epistemic criteria to the extent that he was able to link his evidence explicitly to the claim he made.

7.8 Assessment of Development over the Course of the Unit

As in the previous unit, assessment of the students' work over the course of this unit was undertaken using the criteria standards located in Section 5.7.2. As before, the criteria were organised under two broader categories, Argument Structure and Epistemic Reference. Once again, the primary differences between categories are based firstly on the presence of the criteria component (scores around 1 and 2 are problematic), the completeness of the component (a score of 3 generally means the component exists but is insufficient in some respect), and the extent to which the component approaches ideal for this community (scores of around 4 and 5). Table 7.4 summarises the initial scores for the task and provides a comparison with the final scores.

Table 7.4: Comparison of scores between initial task and culminating task (mean Likert scale score: range 1 (low) - 5 (high))

Item	Indicator	Descriptor Within Community Standards:	Initial Response n=20	Final Response n=20
Argument Structure				
1	Research Question	The research question is clearly and specifically stated	a*	a*
2	Research Question - Context*	The research question informs the wider research context	b*	b*
3	Claim	The claim is explicit, foregrounded and references the question	4.9	5.0
4	Evidence (Grounds)	Evidence reflects audience, is relevant to the research question, contains sufficient (but not extraneous) detail	3.2	4.4
5	Reasoning	Reasoning co-ordinates logically and considers all evidence	3.3	4.0
6	Claim – Context*	Claim implications for wider context are explicit	b*	b*
7	Qualification	Qualifier is provided with details as to when it is applicable	1.2	4.0
Epistemic Reference				
8	Evidence Collection	Evidence collected / generated responds to the question being asked	3.0	4.1
9	Foundation for the Evidence	Evidence provided is data-based (as distinct from fallacy, conjecture, opinion)	4.3	5.0
10	Evidence Gathering	Methodology for obtaining evidence is provided and is appropriate	c*	c*
11	Evidence Organisation / Representation	Representation/ organisation of data are accurate and appropriate for the audience and purpose	2.1	4.2
12	Evidence Interpretation / Analysis	Interpretation / Analysis of evidence meets community expectations: accuracy, clarity, method, efficiency	3.1	3.7
13	Evidence Anomalies or Contradictions	Any anomalous or contradictory evidence is provided and addressed factually or in terms of limitations	c*	c*
14	Reasoning	The justification for making a claim, based on the evidence, is suitable given the community of mathematical learners	2.4	4.1

a* - information that was provided by the teacher and therefore not assessed
b* - no wider context was applicable to this question
c* - the nature of this inquiry was such that these were not relevant

7.8.1 Argument structure

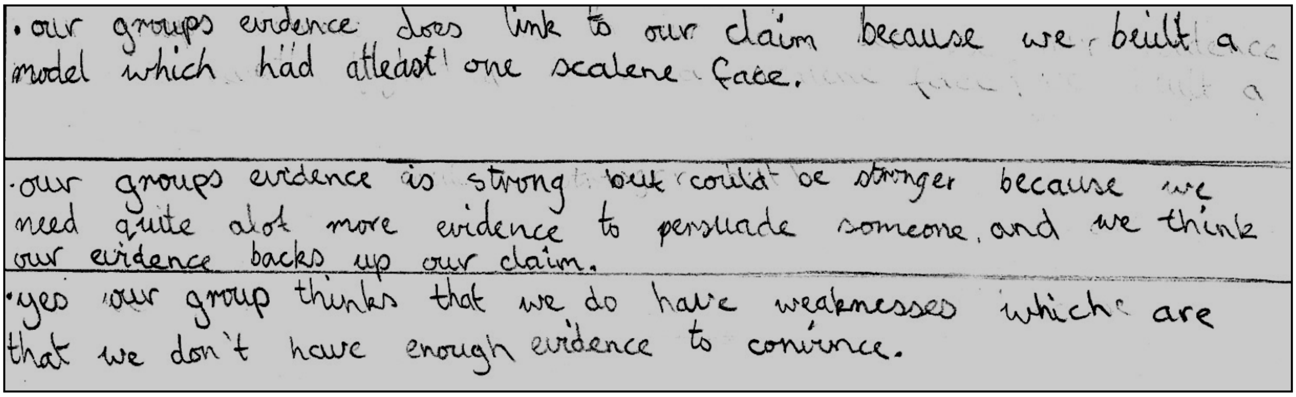
The nature of this Pyramid unit was such that the mathematics was more familiar to the students than in the Barbie unit. As such the teacher took the opportunity to deepen the students' knowledge of argument structure and epistemic acceptability of evidence, rather than focus heavily on the mathematics. If this was successful, gains would be seen particularly across the elements of Argument Structure and this was the case.

Consideration of students' responses as a whole suggests that students had little difficulty articulating a claim at any point during the unit with initial scores already high (Item 3, Table 7.4). This contrasts with the findings in science education and will be addressed in the discussion (Chapter 8). One area of significant gain was in the introduction of qualifiers (Item 7, Table 7.4). This had arisen as a necessary component of the unit when students saw the need for delineating the breadth of their claim. Initially, only one student had attempted to qualify her claim and it was likely incidental: "*A pyramid can have one <u>or more</u> scalene faces*". For the final task, the majority of students had qualified their claim in terms of possible bases, although the language and terminology used may not have been precise as can been seen in Connor's response above (Figure 7.11), where he refers to 'a face this scalene'.

While students were initially quite good at providing evidence, the evidence provided in this unit was largely descriptive in nature (Item 4, Table 7.4). Many of the students chose to say, for example, that they had built a model of a pyramid, rather than providing the pyramid itself. As such, the evidence in the initial phases was insufficient for someone else to use to verify the claim. One interesting issue, particularly in the initial stages, was the students' perceived need for extraneous evidence. In some instances they indicated this was to add detail, or was for their own curiosity; however, in several instances the students expressed a need for more evidence in order to convince others. By the final assessment, most students had determined that one or two well thought out and well-constructed models/nets were sufficient.

The reasoning the students provided was sound in both the initial and the final tasks (Item 5, Table 7.4). In the first instance, students had largely relied on a very general form of reasoning which necessitated some inferring. A sample has been provided to illustrate in

Figure 7.12. Here it is necessary to infer that Laverna is talking about a pyramid with her language rather non-specific. It is necessary to read the reasoning in conjunction with the claim and evidence to fully understand what she is discussing. By contrast, most students made a clearer linking statement between claim and evidence and provided the geometric reasoning, for their final task.



• our groups evidence does link to our claim because we built a model which had atleast one scalene face.

• our groups evidence is strong but could be stronger because we need quite alot more evidence to persuade someone, and we think our evidence backs up our claim.

• yes our group thinks that we do have weaknesses which are that we don't have enough evidence to convince.

Figure 7.12: Initial reasoning sample - Laverna

7.8.2 Epistemic references

While the argument structure considers the components of the argument, and the extent to which they are consistent and co-ordinate, the Epistemic References reflect the quality of the evidence and reasoning in terms of what is considered *ways of knowing* within the mature discipline. While the mature discipline is reflected here, it is important to note that students' responses are considered with reference to what would be ideal for that Community of Learners, that is, what would be expected of developing mathematicians in accordance with the curriculum and appropriate developmental expectations.

The evidence provided by the students in both the initial and the final assessments was consistently based on data rather than their opinion or conjecture (Item 9, Table 7.4). However, in the initial arguments, while it was relevant to the question being asked, the evidence was largely insufficient (Item 8, Table 7.4). This was a reflection of the students' attempts to describe rather than include the evidence they had collected, and was largely rectified by the final assessment although minor errors were not uncommon. The representation of the evidence (Item 11, Table 7.4) was an area of significant gain. The initial representations provided by the students largely involved errors of construction that were significant enough to cast doubt on the claim being made: descriptions based on pyramids that were forced together and nets that would not have created an accurate pyramid.

The reason for the gain was likely two-fold. The first is that the students, through whole-class discussion, became aware of approaches that would enable more accurate representations to be made; and second, that the discussions around lengths and angles both gave the students a measure to test accuracy and indicated to them that such accuracy was deemed important and valued within the learning community. Students cannot be expected to be aware of what to pay attention to within a discipline if they have not been inculcated into that discipline and its values. The understanding of the importance of accuracy was likewise reflected in the interpretation of evidence as students were keen to critique and reject representations which were not sound (Item 12, Table 7.4).

The final item (Item 14, Table 7.4), refers to the reasoning that was advanced to link to justify making a claim based on the evidence to hand. The students were still able to link the representations of the pyramids to their claim of a scalene face; however, in the final arguments presented, the students were able to articulate their reasoning for claiming their shape to be a pyramid, and at least one face to be scalene, along with the mathematical rules (attributes of shapes) that applied. As such, a shift in the reasoning from assumption of underlying mathematical principles, to explicit ones, has been made.

This section has considered changes that were noticeable from the initial through to the final evaluation of the unit on Pyramids. In the following section, the evaluations from both units are presented in order to identify any patterns or scores of particular interest. Several areas were either not scored at all, or only scored on some occasions, due to the nature of the activities and scaffolding required. It would be anticipated that, as students became accustomed to working with argumentation, more autonomy and less scaffolding would be required. However, it is also the nature of some activities that different aspects were to be the focus in response to student development.

7.9 **Evaluation over the course of the research**

The final section to be addressed in this chapter is the nature of the evaluation of Argument Structure and the use of Epistemic References over the course of the research period. While these scores have been discussed individually in each chapter, there are some commonalities worth observing (refer to Table 7.5). The first is in an overall observation of scores in Argument Structure, which more or less climb across the units, and the scores for Epistemic References, which improve within units but drop when the unit changes. Argument Structure of itself changes little between units: the basic structure holds regardless of whether the students are engaged in statistical reasoning, geometrical reasoning, or any other branch of mathematics. It would be suggested then that the students are learning the argument structures and that this knowledge transfers from one setting to the next: at least within mathematics.

The changes across Epistemic Reference were also logical; for example, being able to Envisage Evidence to solve a problem, Representing or Organising Evidence, Analysing or Interpreting Evidence, are all content specific understandings. As students became more experienced in some areas, such as representing data effectively, their domain specific knowledge was enhanced. However, there was little to suggest this would enable them to envisage designing a net for a pyramid. Hence there was a rise within a domain specific task that did not carry to the next task.

Table 7.5: Comparison of evaluation scores over the course of the research (mean Likert scale score: range 1 (low) - 5 (high))

Item	Indicator	'Barbie' Initial Response n=25	'Barbie' Final Response n=25	'Pyramid' Initial Response n=20	'Pyramid' Final Response n=20
Argument Structure					
1	Research Question	a*	2.5	a*	a*
2	Research Question - Context*	a*	a*	b*	b*
3	Claim	a*	2.4	4.9	5.0
4	Evidence (Grounds)	3.6	3.8	3.2	4.4
5	Reasoning	2.9	3.0	3.3	4.0
6	Claim – Context*	b*	2.2	b*	b*
7	Qualification	1.0	1.9	1.2	4.0
	Epistemic Reference				
8	Evidence Collection	a*	a*	3.0	4.1
9	Foundation for the Evidence	a*	a*	4.3	5.0
10	Evidence Gathering	a*	a*	c*	c*
11	Evidence Organisation / Representation	1.9	4.3	2.1	4.2
12	Evidence Interpretation / Analysis	3.4	3.7	3.1	3.7
13	Evidence Anomalies or Contradictions	3.0 (d*)	3.3	c*	c*
14	Reasoning	2.4	3.0	2.4	4.1

a* - information that was provided by the teacher and therefore not assessed
b* - no wider context was applicable to this question
c* - the nature of this inquiry was such that these were not relevant
d* - these results may be misleading as the students could only be assessed on this if their data set included anomalous or contradictory data (n=9 data sets)

7.10 **Summary**

This chapter presented results around the second iteration in the research design: a mathematical inquiry in which students sought to respond to the student driven question, “Can a pyramid have a scalene face?”. The results were once again provided under the wider headings of ‘argument as a product’ and ‘argument as a process’. In considering the argument as a product, the chapter identified some of the students’ experiences in negotiating the Evidence Model and the Argumentation Model. In particular, the students experienced struggles, not this time in envisaging the evidence, but in collecting the evidence they needed through building models and diagrams (representations). In terms of argument as a process, the students developed a further appreciation of sharing knowledge and interacting with others to build deeper understandings and determine ways of overcoming obstacles and improving evidence. The students engaged more deeply this time with the wider classroom community and were far less reticent about questioning and challenging each other to further advance student understanding of what is epistemically acceptable in mathematics learning. In particular, this unit varied from the previous one in terms of context, this unit was not grounded in a context outside of mathematics and there are potential implications in terms of argumentation and inquiry.

In the next chapter, the role of context knowledge will be addressed as part of a proposed model of argumentation which also incorporates argumentation knowledge and mathematical knowledge. Components of these ‘types’ of knowledge identified through the analysis of results will be identified and explained. Interactions between each of the components will also be considered as this offers potential for means of scaffolding and supporting learning. Finally, consideration will be given to those elements which of necessity need to be present in mathematical inquiry-based argumentation: the Signature Elements.

8 Discussion

8.1 Introduction

There has been a growing emphasis on the role of argumentation practices in science education (Driver et al., 2000; Duschl & Osborne, 2002) and research has suggested that the use of these practices can lead students to understand the epistemology underlying the science discipline (S. Simon & Richardson, 2009). Surely then, the possibility for argumentation practices to be extended to mathematics education warrants further research. While some research into argumentation in mathematics education exists, it has focused largely on mathematical proof (see, for example, Conner, 2007; Lampert, 1990) or argumentation as it applies to procedure (see, for example, R. Brown, 2007; Dixon et al., 2009; Forman et al., 1998; Goos, 2004; Yackel & Cobb, 1996). By distinction, the research addressed in this dissertation has focussed on student's argumentation practices while working with ill-defined problems (Anderson, 2002). This has been approached by working with students accustomed in some degree to working with IBL and by then extending the approach to encourage the students' use of argumentation structures and processes. This process has been termed Inquiry-Based Argumentation (IBA).

The aim of this exploratory research was to develop pedagogical theory of IBA in mathematics. In particular, the following questions guided the research:

1. What are key features of an Inquiry-Based Argument model as implemented in a primary (elementary) mathematics setting?
2. What Signature Elements of Inquiry-Based Argument can serve to guide children's mathematical argumentation?

This research was approached by adopting a Design-Based Research methodology (Cobb et al., 2003; Lesh, 2002) to support the creation of theory built on intervention and reflection. Two full cycles of intervention were implemented with a single class of middle primary students (aged 8-10 years) who worked with the author. These interventions consisted of two extended mathematical inquiries of approximately 15 hours each. The first centred on proportional reasoning and statistical inference through addressing whether Barbie could have human proportions, and the second took a focus on geometrical reasoning (as both context and mathematical focus) to determine whether a scalene-faced pyramid was possible. In each inquiry, students' practices were extended to address epistemic argumentation (Toulmin et al., 1984); that is, they were guided towards taking a claim-evidence-reasoning (McNeill & Martin, 2011; Zembal-Saul et al., 2013) approach to responding to the inquiry questions, with a focus being the use of discipline-based evidence to make and justify claims.

Design-based research responds to a specific context and thus it should be specified from the outset that the findings here are exploratory, specific to the context presented, and are by no means suggested as being transferrable to other contexts. Despite the humble nature of these findings, the possibility for such theories to provide insight and to act as a conceptual beginning for future work justifies their undertaking. It is hoped that the work presented in this chapter will serve, not to provide 'answers', but to be a starting point for a framework of research and discourse addressing the potential and practicalities of IBA in mathematics.

In this discussion, the developing theories from this study will be presented, including a tentative model of Inquiry-Based Argumentation in Mathematics (Sections 8.2 – 8.6) and proposed Signature Elements of Inquiry-Based Argumentation practices (Section 8.7) in primary mathematics.

8.2 A Model of Inquiry-Based Argumentation in Primary Mathematics

The first research goal was to identify some key features of an IBA model as implemented in a primary (elementary) mathematics setting. Establishing a model of mathematics argumentation in education would serve to "contribute to a theory of learning that can capture and convey the essential features of the learning environments that we design" (A. L. Brown & Campione, 1996, p. 290).

The primary purpose for proposing such a model is the potential it offers for conceptualising and discussing, or capturing and conveying, those essential features of IBA. By conceptualising IBA, researchers and educators have an increased opportunity to visualise what it entails, and be cognisant of components for planning or evaluation purposes. Development of a model also potentially promotes a common language around which to articulate discussion and critique around the features of IBA.

Prior to conducting the research, and based on a broad understanding of the literature, a tentative impression of context knowledge and mathematical knowledge coming together to feed into the development of argumentation knowledge was envisaged (Figure 5.2). Open-coded data were organised into common components which were then aligned to one or more knowledge domains (mathematical, contextual and argumentation) (see Section 7.7, level 2). So for example, material coded as impacting on students' affect was related to both their mathematical knowledge and engagement with mathematical aspects of the inquiry, and with the context itself. Thus '*affect'* is assigned as a component of both '*Mathematical Knowledge'* and '*Context Knowledge'* domains. This provided the skeleton of the model and also some insight into the roles of both the domains and the components of the domains by examining the material coded to each component.

One unexpected insight was the extent to which each domain served to support others. The support of the context in the Barbie unit for developing the students' ability to visualise proportion was, for example, one way in which Context Knowledge clearly supported Mathematical Knowledge. Prior to the analysis being undertaken, the impression had been that Context Knowledge and Mathematical Knowledge would feed together to develop the argument and the Argumentation Knowledge; however, this was not the case. In many instances, the students' developing understanding of argument structure led to their seeking stronger or more defensible evidence. Thus, Argument Knowledge became one of three correspondingly prominent knowledge domains as is reflected in the model proposed presented here (Figure 8.1).

This model incorporates each of three domains that the students appeared to be drawing on during their involvement in the learning sequences. These have been named *Context Knowledge*, *Argumentation Knowledge* and *Mathematical Knowledge*. Figure 8.1 illustrates each of these domains and identified components that contribute to the domain. It is not anticipated that this list would be exhaustive, but it does provide a start to developing an overview of IBA and encompasses elements to be discussed in this dissertation. Other components, such as engagement, disposition, interpersonal knowledge, and classroom culture require acknowledgement; however, they were outside the focus of this study.

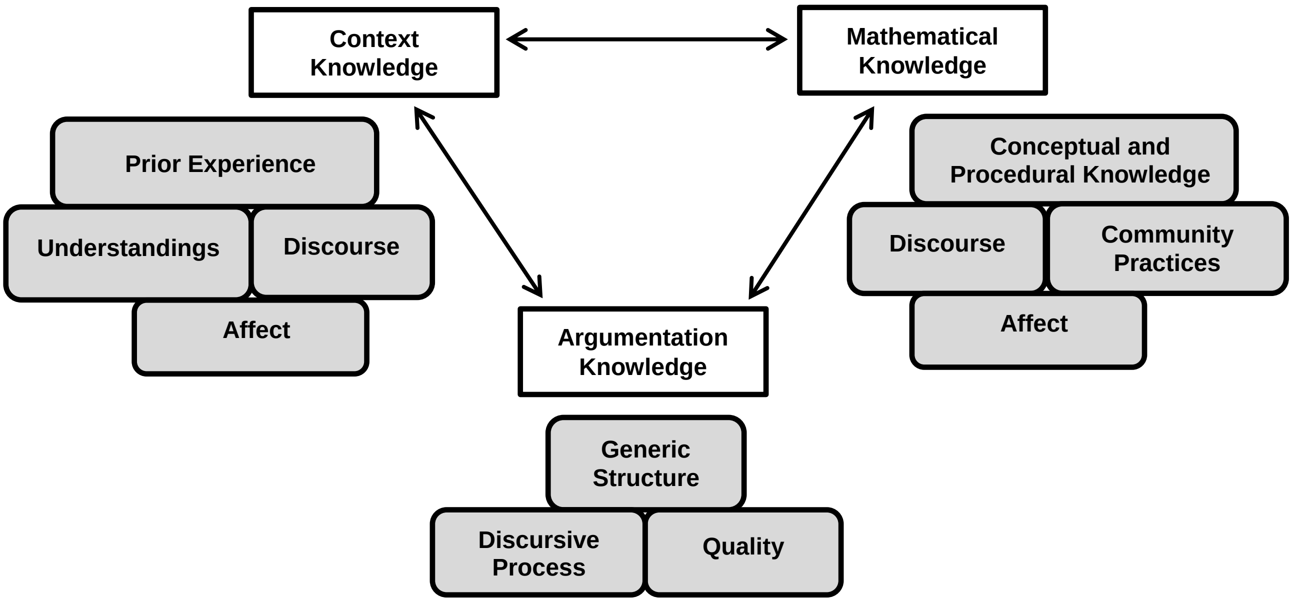


Figure 8.1: Model of interacting domains in Inquiry-Based Argument.

In the Inquiry-Based Argument units with which the students in this research project engaged, there appeared to be three bases of knowledge from which they were required to draw: the mathematical knowledge required to progress through the question; the understandings that surrounded the context employed; and the knowledge of structures and conventions of argumentation. At any one time, students could be drawing on one or more of these bases. In order to clarify the nature of these domains, this section will provide an overview of each domain in order to consider later the role that each domain played and how, in turn, these domains interact at times to support students through the Inquiry-Based Argument process.

8.3 Components and Role of Context Knowledge

Examining the role of context in the two teaching units suggested four components of Context Knowledge: *prior experiences* with the context, *understanding* of the context, *affect*, and context *discourse*.

8.3.1 Prior experience

Prior experience with the context describes the familiarity and opportunities for past engagement that the students had with the context at hand. These experiences may be vastly different for students and result in significantly different understandings and affective responses. They may or may not be school based or culturally specific, but they would likely be different from student to student. For example, in this instance, all the students in the class were familiar with Barbie, so much so that they were providing the teacher details of Barbie's fictitious background. However, Barbie is an American doll and ostensibly associated with the values of American culture. This context would be less well-known to students of non-Western cultures and alien to those of Middle-Eastern cultures (where the Fulla doll has Barbie status but is far more moderate in terms of body proportion and less sexualised (Wikipedia, 2013b)). Had the class contained students that had not been raised predominantly in Australia (or another Western nation), the context may have been confusing, and lack of prior experience may have served to distance and exclude the students from the discourse.

8.3.2 Understanding

A second and related component of context knowledge is the *understanding* of the context. While this may be experientially developed, it is not necessarily so. Understanding can be obtained through teaching, reading, or observation for example. Understanding has been used distinctively from *prior experiences* by a simple distinction: prior experiences have been considered as those occurrences that are not open to change as the experiences are historically bound; however, understandings, which may be built on prior experiences, are able to be challenged, deepened and altered through the application of a reflective lens.

8.3.3 Affect

Affect is an area of high importance in mathematics, at least in Australia, as students have demonstrated increasing reluctance to engage in mathematics at higher levels (McPhan et al., 2008). Prior or ongoing experiences and understandings in turn may instil and elicit emotive or affective responses in students that are evident in response to the context. Students' prior experiences may be the source of particularly strong feelings or intuitively held beliefs about a context and these may serve to influence the students' engagement, approaches, or interpretation of the results.

8.3.4 Discourse

The final component of Context Knowledge to be addressed is the discourse of the context. Students' familiarity with the context, and this may well be linked to prior experience, has the potential to provide an underlying language and terminology for use. While this is not essential to the mathematical understanding, it has the potential to support or make more difficult the students' discursive involvement.

By way of illustration, when the students engaged in ill-structured inquiry, the questions they addressed were situated within either a non-mathematical context (Barbie) or a mathematical context (Pyramids). The difference being that, in the former instance, while mathematics was applied to address the question, there was also a non-mathematical component. The non-mathematical components being the *prior experience* of a Barbie doll: the nature of dolls as a child's plaything that is representative of humans in some form and any experiences that may surround the students' previous engagement with Barbie. Such prior experiences may have an *affective component*: a degree to which the student attaches an emotional response to the context; for example, whether the student enjoyed playing with Barbie as a child, or whether the student's parents sent a strong message about the appropriateness or otherwise of dolls as role models for children. These experiences may have fostered *understanding*s around the concept of human proportion, role models, marketing influences on children and so forth, along with any associated *discourse*. By contrast, these components were somewhat different in the Pyramid example. In terms of *prior experience,* the students had some surface familiarity with the concept of Egyptian pyramids; however, it is unlikely that this would create a strong *affective* response. Prior study of geometry would have resulted in the students developing both *understandings* and *discourse* around the topic, although this is more correctly Mathematical Knowledge rather than Context Knowledge.

8.3.5 **The role of context knowledge**

The overview of components of Context Knowledge is brief as the dissertation audience is likely familiar with these ideas. Of greater focus is the role that the Context Knowledge plays in Inquiry-Based Argumentation: serving to situate the application of mathematics and authenticating the learning; serving to engage the students in the learning sequence; and, providing a scaffold with which to support learning. The first two will be addressed here with the latter addressed when interactions between Mathematical, Context and Argumentation Knowledge are addressed in Section 8.6.

Situating the application of mathematics and authenticating the learning

In the two learning sequences examined in this dissertation, the contexts are significantly different. The context developed to address the first unit explicitly made connections for the students between aspects of mathematics (fractions, proportion, and statistics), other areas of the curriculum (human proportion in the Arts, media influence on health) and the 'real' world (clothing manufacture). Such connectedness is a goal identified by the United States' National Council of Teachers of Mathematics (1991) as a remedy for mathematics being seen as a body of isolated concepts. The embedding of units in contexts that enable a coherent, and perhaps explicitly made connection beyond the more formal uses, has the potential to address student disconnectedness (McPhan et al., 2008). As identified in the introduction, Australian students are discontinuing higher-level, enabling mathematics subjects (Australian Academy of Science, 2006; Barrington, 2011, 2012, 2013) with research indicating that a significant reason was the students' perception that mathematics bore little relevance to the 'real' world (McPhan et al., 2008).

Not only does embedding learning in a context-rich problems have the potential to ensure mathematics is seen as connected and useful, but it also has the potential to more deeply inculcate students into the situated context of the application:

> Just as carpenters and cabinet makers use chisels differently, so physicists and engineers use mathematical formulae differently. Activity, concept, and culture are interdependent. No one can be totally understood without the other two. Learning must involve all three. Teaching methods often try to impart abstracted concepts as fixed, well-defined, independent entities that can be explored in prototypical examples and text-book exercises. But such exemplification cannot provide the important insights into either the culture or the authentic activities of that culture that learners need. (J. S. Brown et al., 1989, p. 33)

Conversely, the Pyramid unit, building on a topic selected by students, lacked authentic context links to the 'real' world as such. However, it more closely addressed the way in which pure mathematicians might work by focussing on a problem that was both mathematically situated and without an immediate, known application. This problem may have had no identifiable use beyond seeking to determine a deeper understanding of a topic. However, it furthered the collective understanding of the community (Scardamalia, 2002) and enabled the students to posit their own theories about mathematical concepts: both considered goals for Knowledge Building and both authentic goals of pure mathematics.

Engaging the students in the learning sequence

Another role of the context was that of engaging the students in mathematical learning. The Barbie unit could have been potentially disengaging for male students; however, this did not appear the case, although several of the boys initially expressed a preference for measuring Batman's proportions. The students spent some time talking about the unit purpose and reflecting on reasons why 'normal' proportions for humans would be required, including forensic applications and furniture design (not reported in the results). Students were thus able to see important reasons for establishing human proportions that were connected to the real world. Students can be engaged by tasks they see as having value (Fielding-Wells & Makar, 2008b) and this further confirms the essential nature of utility in task design (Ainley et al., 2006).

The Pyramid unit cannot be thought of as completely decontextualized, as there were potential, yet distant conceptual links; for example, students having seen pictures of pyramids in Egypt. However, the context here was predominantly situated in geometry, with prior knowledge coming from this field. The context addressed was one that the students proposed and continued interest was demonstrated with the class members bringing in sample pyramids they had constructed at home of their own volition. Disengagement in mathematics has long been a concern in Australia and has been an area recommended for specific focus (McPhan et al., 2008), so this potential for engagement would appear to be a critical role of the context. However, it would appear that while a specific non-mathematical context is not essential to creating interest, it certainly could be a significant factor.

8.4 Components and Role of Argumentation Knowledge

The second domain of the proposed model is *Argumentation Knowledge,* which draws upon the knowledge the students have around argument structure and argument process.

In this section the nature of argument as it is incorporated within the model will be addressed before discussing the role of Argumentation Knowledge. Analysis of the classroom activities and interactions, along with consideration of prior literature in this area, suggests several components of Argumentation Knowledge significant to IBA: *generic structure* of argument, *discursive process* of the argument, and the *quality* of the argument. These components of Argumentation Knowledge will be discussed individually before addressing the role played.

8.4.1 **Generic structure**

During the Pyramid unit, the students negotiated and developed a model of Inquiry-Based Argument for use in the classroom which they thought would be useful to support the process for others. The students' model is included at Figure 7.9 (reprinted here as Figure 8.2) as the final model the students agreed upon. It shows the Inquiry focus of Purpose-Question-Evidence-Conclusion while expanding the Conclusion to encompass the generic structure of an argument: claim, evidence, reasoning and qualification.

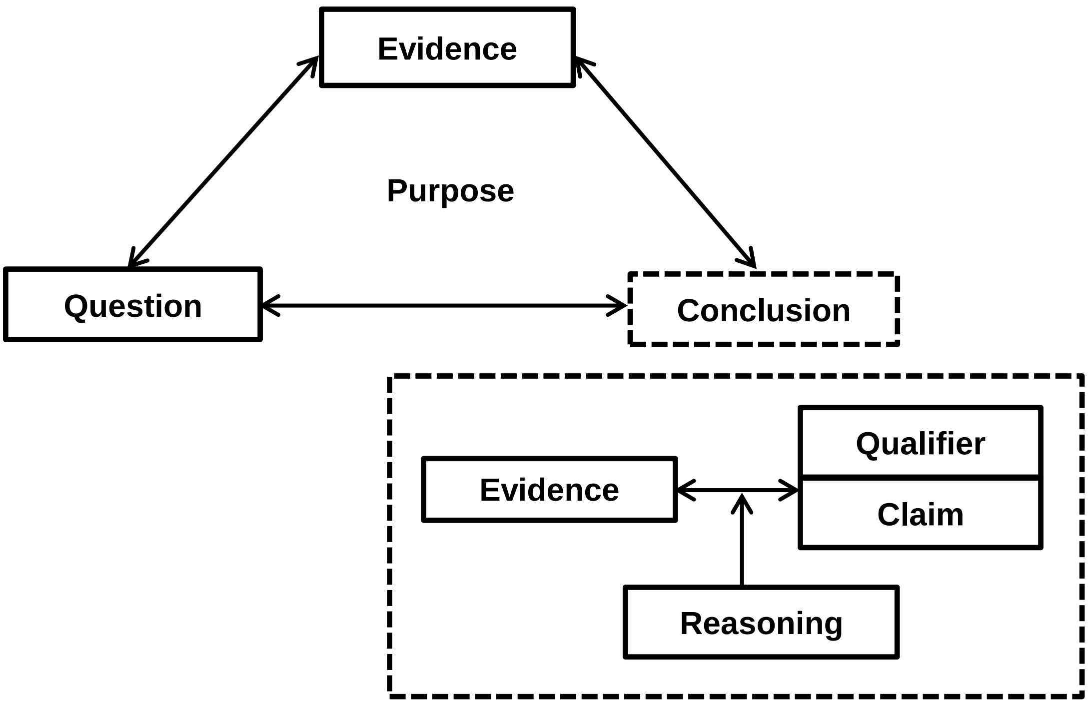


Figure 8.2: Essential elements of Inquiry-Based Argument

In order to monitor students' progress in developing understanding of the generic argument structure, two sources of evidence were used: the Argument Structure rubric

(Table 5.6) which was used to assess students' work samples to identify change over time and experience; and evidence of change from students' contributions during classroom discussions.

Students' written arguments were examined and scored against the rubric for Argument Structure across seven categories, of which three were not addressed sufficiently to enable progress tracking of the students' development. However, these may remain important indicators for more advanced studies. The four remaining categories were addressed in this research design and these categories are discussed in detail below: Claim, Evidence, Reasoning, and Qualification.

Claim

Analysis of student discourse and artefacts suggested that there are three important aspects of claim in students' mathematical arguments:

1. That any claim is derived from (mathematical/statistical) evidence;
2. That the claim is articulated clearly; and,
3. That the claim provides sufficient, accurate detail.

An essential aspect of epistemic argumentation is that any claim be derived from evidence. As Epistemic Argument aligns closely with Knowledge Building, the aim of the argument is to develop the best understanding of the situation/phenomena possible rather than a 'winning solution' and so the arguer seeks to put forth a claim that is *derived* from evidence, as distinct from using evidence to *support* a claim (Biro & Siegel, 1992; Lumer, 2010; Siegel & Biro, 1997). Once the claim is determined, it should be clearly provided. In this instance, the students were consistent with their provision of claims throughout; although, these often lacked detail in that they often did not appropriately reference the question but rather the broader purpose or context. This has the potential to be problematic given that ill-structured mathematical questions require refining, and the audience needs to be aware of what decisions were made and what the breadth of the claim is. In the case of mathematical argumentation, this need to define parameters appears best taken on by the use of a qualifier.

Qualifier

Toulmin (Toulmin et al., 1984) explains the necessity to qualify claims in order to express a degree of certainty in the claim. The qualifier can take on several roles, it can:

1. Express the strength of a claim in terms of modal qualification - probably, certainly, possibly (modal qualifier); and,
2. Limit the condition that the claim applies to (delineating qualifier).

While modal qualifiers have a use in all argument types (Toulmin et al., 1984, p. 90), they would appear to be potentially useful in arguments involving school-level statistics. Younger students are sometimes introduced to informal statistical inference. Informal statistical inference requires the use of non-deterministic language to express a level of uncertainty about any population-based inferences made when working with sample data (Makar & Rubin, 2009). So this lends itself to qualification through the use of probabilistic language. In this study, students most commonly expressed modal qualification verbally through unwillingness to be deterministic about the normal human range. One example came from Geneva in her oral presentation of her argument: “The range is, well the most, I said the normal range *could be* from 1.0 to 3.5, but *I’m not completely sure* about the 3.5 part. Because 3.0 was the outlier”. However, these were not expressed in the written presentations. One way in which students may have implied modal qualification was through the use of the phrase “I think” used as “*I think barbie dus not have human poportions for this meserment”* (Oliver, Figure 6.7). Whether this phrasing was intended to qualify the claim or is merely a turn of phrase is unclear. Despite the first unit being based on a sample, very little spontaneous use of modal qualification was identified: possibly through limited understanding of the differences between population and sample data (Pratt et al., 2008). Certainly this is an area that required further research as the role of qualifiers may be unique and of importance when considering informal reasoning in statistics.

The second type of qualifier incorporated is tentatively termed ‘delineating qualifiers’ as their role is to identify limits to the conditions that the claim applies to, and these limits may be contextually situated. The nature of ill-structured questions is such that the questions need refining and negotiating to a point at which they are researchable and this is of itself limiting. Thus, qualifiers may provide a way of expressing the limitations surrounding the inquiry and impacting on the argument: For example: <u>*Based on the proportions of a non-random sample of 26 adult females*</u>*, Barbie’s proportions fell outside the range of a normal human.*

Thus the students were encouraged to address this idea of qualifiers and incorporate these delineating qualifiers as required. Students had very little trouble in doing so although their language clarity required addressing on occasion (Item 7, Table 7.5: student mean scaled scores moved from an initial response of 1.2 to a final response of 4.0) The final qualifiers were predominantly well stated and unambiguous, for example: "A pyramid *can* have a scalene face *when it has an irregular pentagon or a triangular base*" (Andrea). In Andrea's example, the use of 'can' has been used as a modal qualifier. A pyramid does not *have* to have a scalene face under these conditions but it is possible.

The CER model (McNeill & Krajcik, 2011; McNeill & Martin, 2011; Zembal-Saul et al., 2013) does not incorporate qualifiers explicitly, although this would not preclude them from being built into the claim as a qualified claim. Given the ambiguous and ill-structured nature of inquiry problems (Anderson, 2002), the addressing of modal and delineating qualifiers would appear valuable. It would seem that any model of mathematical argumentation, based on ill-structured inquiry, must necessarily incorporate qualifiers at the more advanced level.

Evidence

In presenting an argument, the evidence provided would ideally:

1. Be sufficient to justify the claim made (without excessive extraneous information);
2. Reflect awareness of the audience in the presentation or delivery; and,
3. Enable the addressing of the inquiry question.

In IBA, evidence must be sufficient to support the making of the claim. This was considered by Sampson and Clarke (2006) as an important criterion for a quality scientific argument, but one that was problematic in school-based scientific argumentation as students were previously reported to have difficulty recognising too little evidence (Zeidler, 1997). By contrast, in this study (in both the Barbie and the Pyramid units), students demonstrated a desire to include as much evidence as possible, including duplicated information, thus privileging or valuing quantity over quality (Lines 71-80, Section 7.2.3). Two approaches served to focus the students effectively on quantity versus quality of evidence: the use of classroom discussion to address the topic explicitly, and a strong focus on audience. This consideration of audience with the students was motivated by a desire to convince others of the strength of their evidence most effectively.

The final aspect of evidence considered was the extent to which the evidence related to the research question: again an assessment focus identified by Sampson and Clarke (2006). While the students in this research project had little difficulty with obtaining *relevant* evidence (other than any limitations in their mathematical knowledge), they were relative novices and were working with teacher guidance. Students working autonomously would likely have greater potential to collect evidence not directly related to the question. While Zeidler (1997) did not report this as problematic in his research, it is unclear to what extent the students in Zeidler's study drew conclusions or were supported and directed towards obtaining appropriate evidence. A student focus on the provision of reasoning may well identify shortcomings in evidence as students attempt to justify their claims based on the evidence.

Reasoning

In the CER model (McNeill & Martin, 2011), the role of reasoning is to show why the data counts as evidence to support the claim and therefore should include appropriate discipline-based principles. With the epistemic argumentation approach taken in this study, reasoning rests on the epistemological acceptability of the evidence, the appropriateness of the claim in terms of the evidence, and the explicit claim-evidence link. Therefore, reasoning encompasses the mathematical content, procedures, practices, and discipline norms which enable the arguer to co-ordinate evidence and claim: the reasoning brings in the mathematics knowledge or mathematics theory.

By way of illustration, in the Barbie unit, the informal mathematical reasoning of the students involved rule of thumb ideas about deciding on the population based on their sample. Students 'eye-balled' the distribution and made a decision based on how distant any outliers were as to whether they were 'normal' or fit within the distribution. Essentially the students were considering aspects of distribution such as mean, spread, and standard deviation; however, they were doing so informally and at a level commensurate with their development. Hence, this is the mathematics (or statistics) that enables the students to make a call about Barbie being human or not. The defending of mathematical practices here has potential to engage the students critically with mathematical knowledge and to be a source of deep understanding.

In conclusion, the generic structure component of Argumentation Knowledge as implemented here manifests the same elements of Claim-Evidence-Reasoning as the CER model that it was theoretically drawn from (McNeill & Krajcik, 2008, 2011; McNeill & Martin, 2011; Zembal-Saul et al., 2013); however, it incorporates an additional element of qualifiers. The role of the Qualifier appears significant in delineating the parameters of the claim, or identifying likelihood or certainty, and would warrant further research.

8.4.2 Discursive process

Discursive Process is the second identified component of Argumentation Knowledge. Discourse is an integral component of the mathematics argument model as argument is essentially a discursive practice, even if one is arguing with oneself. However, it is not a practice which is usually encountered in the learning environment, and hence a culture of argument needs to be developed in the classroom (Pontecorvo & Pirchio, 2000). To work as a community of Knowledge Builders, or engage in epistemic argumentation, requires a degree of discursive adroitness.

Before the implementation of IBA in this classroom, the students were already familiar with IBL practices in mathematics. This partly entailed the students being immersed in a culture of evidence-based learning, with students often challenged to explain their thinking and reasoning to the class. Perhaps the most unusual aspect of this discourse was that the teacher would challenge correct and incorrect answers alike. This was important in that the students did not cue that responses were incorrect on the basis of being challenged by a teacher, and came to value justification as an integral part of a response to a question. Nonetheless, while the classroom culture was such that students were accustomed to being challenged, they were still accustomed to that challenge coming from the teacher, rather than fellow students. It was deemed important that students learnt to provide and accept challenge appropriately.

In particular, students needed to be taught to focus on the argument itself rather than the ‘owner’ of the argument. Students needed to accept that any challenge was not to them personally, but to the group’s collective knowledge, in order to advance understanding –

hence why a Knowledge Building focus was integral to the advancement of argumentation. In the initial stages of the Barbie unit, it was noted students were quite comfortable providing anonymous feedback. However, when students presented their arguments ‘publicly’, and the student audience was invited to provide feedback or question the evidence, they were reluctant (for example, see Section 6.5). To facilitate the discussion, the teacher elected to model questions and engage in explicit instruction about appropriate and inappropriate questioning. Unsurprisingly, this concurred with the findings of Jimenez-Aleixandre and Erduran (2007), who contended that explicit instruction, modelling and task structuring were essential to teaching argumentation discourse in science classrooms (see also, Pontecorvo & Pirchio, 2000; S. Simon & Richardson, 2009: refer to Section 3.6.3). The students undertaking the Barbie unit responded well to this form of support, and within a short space of time began to mimic the teacher’s modelled questions. This reluctance was not seen again despite over eight months between the first and second units, with the students challenging aspects of others’ evidence early in the Pyramid unit.

The willingness to challenge other students’ ideas and evidence may have been further facilitated by the Knowledge Building emphasis in the classroom. The underpinning principles of *Knowledge Building Discourse* and *Improvable Ideas* (Scardamalia & Bereiter, 2010: refer to Section 1.3.2), while not named as such to the students, had slowly become a part of the classroom culture. This may have afforded students the perceived right to be critical of evidence as something which is external to them personally; students’ avoiding of ‘ownership’ of information, findings, data, or evidence is important.

8.4.3 Quality

The quality of the argument is an important aspect of this work. Focussing students on the importance of a quality argument enabled them to see the importance of the role of evidence: both in helping to support a strong claim, and it making such a claim in the first place. While the development of the Epistemic References rubric enabled the researcher to focus on evidence quality, the students were engaged in multiple discussions to this effect; particularly in the second unit on pyramids, when the students had grown accustomed to argument structure. A sample argument was presented to the students with successive examples of flawed evidence and reasoning and this served to draw students’ attention to the need for quality evidence which was reliable and valid. Eventually the students engaged in a lengthy discussion about the various roles of evidence (Section 7.2.2).

Evidence quality is integral to epistemic argumentation. Section 8.7.1 addresses the epistemic basis of mathematical evidence and reasoning in greater detail and the reader is referred to this section.

The identified components of Argumentation Knowledge have been addressed in this section, with generic structure referring more directly to the argument product, while the discourse reflects more the argument process. However, these are not dichotomous as the knowledge of generic structure is required to engage in argumentative discourse and discourse leads to improved quality of the argument product. The role of Argumentation Knowledge will be addressed in the following section and the nature of the interactions of these components will become more apparent.

8.4.4 The role of argumentation knowledge

The generic structure of the argument provided a framework which supported students to construct an evidenced conclusion, and provided a process whereby students could be supported and increasingly challenged to present their evidence, draw a claim from the evidence, and then justify their reasoning. Aspects of structure have been discussed above and so the process of extending argumentative goals in the class will be addressed here.

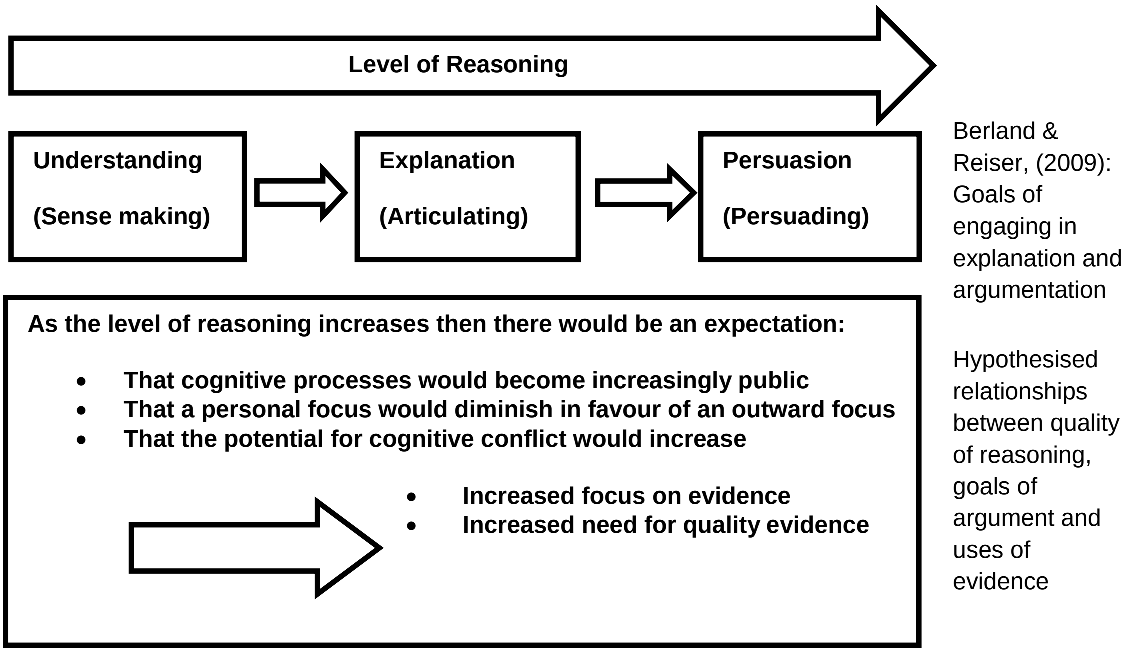


Figure 8.3: Hypothesised model of potential interactions between reasoning, goals of argumentation and use of evidence.

A hypothesised model, drawing on Berland and Reiser's goals of argument (2009), was presented in the theoretical framework (Figure 4.4 and reproduced in Figure 8.3). The expectation was that as the level of reasoning increased, there would be a diminished focus on personal understandings, beliefs, and reliance on internalised sources of knowledge. This would come about through the increasingly public nature of a student's cognitive processes which in turn would enable those cognitive processes to be identified and challenged: potentially increasing the reliance on externalised, objective, defensible evidence. Each of these expectations will be addressed individually.

Cognitive processes become increasingly public

As students shift from their own internalised understandings towards explaining and even justifying their understandings to others, their underlying thought processes become visible to others through the discourse involved. It is possible for students to present evidence without explaining the processes that led to the evidence, as in rhetorical communication (van Eemeren & Grootendorst, 2004; van Eemeren et al., 1996); however this omits articulation of the reasoning, which is the centre of deeper mathematical understanding.

Three specific instances were noted when students' cognitive processes were particularly well revealed: class discussions, small group discussions, and presentation of arguments.

During class discussion time, students were provided with opportunities to discuss ways in which they could gather and represent evidence, apply mathematics, interpret evidence and so on. An example has been provided in the results which illustrates the extended conversation around the relative merits of tallies versus dot plots for representing data (Lines 51-75, Section 6.4.2, second iteration). During these discussions, students proposed ideas and provided their reasoning for the ideas, enabling insights into their thinking.

The second opportunity to identify student thinking arose during group work when the students worked together to negotiate how to obtain evidence, use evidence, and represent and interpret it. Again students could propose and challenge ideas, build on them, defend them and so on. Students who were less willing to make comments in whole class discussions were often more open in smaller groups and this became an opportunity for them to be heard directly: even when the teacher was away from the group, the members served to act as an audience. An illustration of such group discussion is illustrated in Section 7.2.4 (Lines 54-70), when students in their groups were first brainstorming potential sources of evidence for their pyramids. What is not evident from this group discussion was the diversity of these students: one was a very dominant student, identified as gifted, while another was very reticent about communicating, identified as being on the Autistic Spectrum. So while a limited, single example, this conversation suggested that students were adopting more of the *Community Knowledge* focus with ideas considered *Improvable* (Scardamalia & Bereiter, 2010; Zhang, Hong, Scardamalia, Teo, & Morley, 2011). The transcript suggests that the students were accepting of each other's ideas but willing to critique them also.

Potentially, the greatest opportunity for understanding cognitive processes came during the presentation of the argument. As the student or student groups presented their claim, evidence and reasoning, there was a valuable opportunity for students and teachers to probe understanding and thinking quite deeply. Perhaps the best example of this was the probing of Geneva's responses as the teacher and researcher tried to determine whether Geneva's extremely wide spread of estimated human female proportions demonstrated a lack of understanding of proportion (Lines 138 – 152, Section 6.5). When challenged however, Geneva argued quite well that it was possible to have very short legs and be very tall if you have had some sort of accident to your legs; however, she conceded that she should check the data for errors also.

Aside from the insight these discussions gave the teacher into the cognitive processes of the students, the other students also benefitted, as children who were not willing to articulate their own ideas were involved in the learning peripherally: potentially hearing ideas that challenged their own thinking through involvement in the responses as an audience member (J. S. Brown et al., 1989; Lave & Wenger, 1991).

Potential for cognitive conflict increases

The public nature of students' thoughts, ideas and understandings paves the way for challenge to students' own and others' ideas and provides a means to develop the cognitive conflict to create the disequilibrium necessary to move students to seek new understanding (Harel, 2008; Harel & Koichu, 2010). Three specific provokers of cognitive conflict were identified as a result of this study: conflict brought about by the teacher; conflict brought about by students; and, conflict brought about by the task.
The first, conflict brought about by the teacher, is fairly self-explanatory. Students' discussions offered multiple opportunities for the teacher to question the students, challenge them for their evidence or reasoning, sow seeds of doubt, present 'what-if' scenarios or extremes, and to manipulate or establish parameters to the inquiry task to incorporate conflict.

The second is conflict brought about by the students, and this in many respects appeared more effective in engaging extended dialogue. Suggested reasons for this might include the increased level of comfort students felt over arguing with each other rather than with the teacher, or there may have been a perception that the teacher would be right and therefore shouldn't be challenged. To illustrate, while many of these student challenges occurred in brief exchanges, one of the strongest driving forces for the Barbie unit was a friendly disagreement between two girls over whether Barbie's neck was disproportionately long (Lines 22-28, Section 6.3.2). This led one student to suggest testing the doll, which acted as a springboard into planning the inquiry.

Finally, the task itself on occasion acted as a source of conflict, with a certain amount of feedback inherent in the nature of the task or the approach to the task. During the Pyramid unit, Delmar became quite pleased when he determined that each pair of adjacent sides on the net of the pyramid (that fold to become an edge) must be the same length. He then suggested that if all the pairs of adjacent sides on the net are the same length, it should always form into a pyramid (Lines 92 – 99: Section 7.3.1). When Delmar and the other class members went on to test this theory they were surprised to discover that it still didn't create a workable net for a pyramid. Thus the task itself created the conflict necessary for the class to then consider there must be another factor: the angles on the triangular faces. The role of the task and power of the task itself to cause cognitive conflict suggest that more research into task design similar to that of Ainley et al. (2006) would be valuable.

Personal focus diminishes in favour of outward focus

The reference here to personal focus is to that of own knowledge; that is, a focus on developing one's own understanding at a sense-making level. A hypothesis was made that, as students began to focus on explaining their understandings and convincing others of such, there would be a shift from internalised knowledge towards structuring of thoughts and ideas to provide to *Collective Community* knowledge inherent in Knowledge Building (Scardamalia, 2002).

There are two important aspects to this: the first is that students, in shifting from internalised understanding to outward demonstration, are required to engage deeply with concepts in order to coherently articulate them, and even more deeply to defend and justify them. The second is that through externalising knowledge, students are contributing to a collective pool of knowledge in the classroom. Positioning all students as contributors and valuing contributions takes the onus for production of knowledge off the individual student. This could be a critical factor in mathematics education in terms of self-efficacy, confidence and engagement.

The shift to an outward focus would be anticipated to take time as it would require a change in classroom culture (Zhang, Scardamalia, Reeve, & Messina, 2009). Students would also be anticipated to have initial difficulty in envisioning what would help to convince others and, like the students in this study, demonstrate a tendency towards an egocentric viewpoint: expressing views that people would believe them if they put enough information, or if the audience was able to understand them (Table 6.6). However, the students were able to shift, relatively quickly, to more outward-focussed approaches. The transcript of one group was included in the results, and this was by no means a unique group, to demonstrate the group's multiple references to the evidence needed *to convince* and to have the audience *believe* (Lines 74-83: Section 7.2.4). On this occasion, the students' imagined evidence was solely constructed around envisaged, and carefully thought out evidence (Item 9, Table 7.1). Ostensibly, the students' focus on the explanation, and more specifically, the convincing of others, was tied to their search for evidence and more convincing ways of representing the evidence to the audience.

Increased focus on evidence and evidence quality

The model of hypothesised interactions (Figure 8.3) suggested that: personal focus would diminish in favour of outward focus; cognitive processes would become increasingly public; and the potential for cognitive conflict would increase. The model was predicated on an expectation that these three aspects would necessitate an increased focus on evidence and on evidence quality. This was only partially correct, as it would more accurately be described as a complementary and potentially reciprocal process: the discourse involved in planning, seeking, and interpreting evidence resulted in deeper understandings becoming public, which in turn provided further potential for conflict. This was noted in both units. During the Barbie unit, students' data collection, recording and interpreting of the human range for particular proportions led to some students' emerging understandings about population data versus sample data. While some students were satisfied with expanding their range of scores slightly to acknowledge that there would be other scores in the population, other students became conflicted when publicly challenged over where the scores could reliably sit. This resulted in a student revisiting the evidence to explore it further and improve its quality through checking accuracy (Lines 128 – 152, Section 6.5).

A reciprocal interaction between evidence/evidence quality and public cognitive processes/cognitive conflict was observed in the Pyramid unit. As students created their pyramids and presented them as evidence to the class, the inaccuracies of construction began to cast doubts in the minds of students. As a result, students explored the geometric properties of the triangles further to determine that the angles must total 180 degrees. Thus they used this knowledge to critique representations (including their own) and to improve the quality of evidence. Again, there was a sequence where evidence, presented publicly, resulted in conflict around the quality of the evidence and served to improve subsequent evidence (Lines 150-162, Section 7.6.2 provides a broader illustration of this interaction).

These hypothesised relationships between quality of reasoning, goals of argument and uses of evidence, may be better left as resulting processes that are considered interlinked

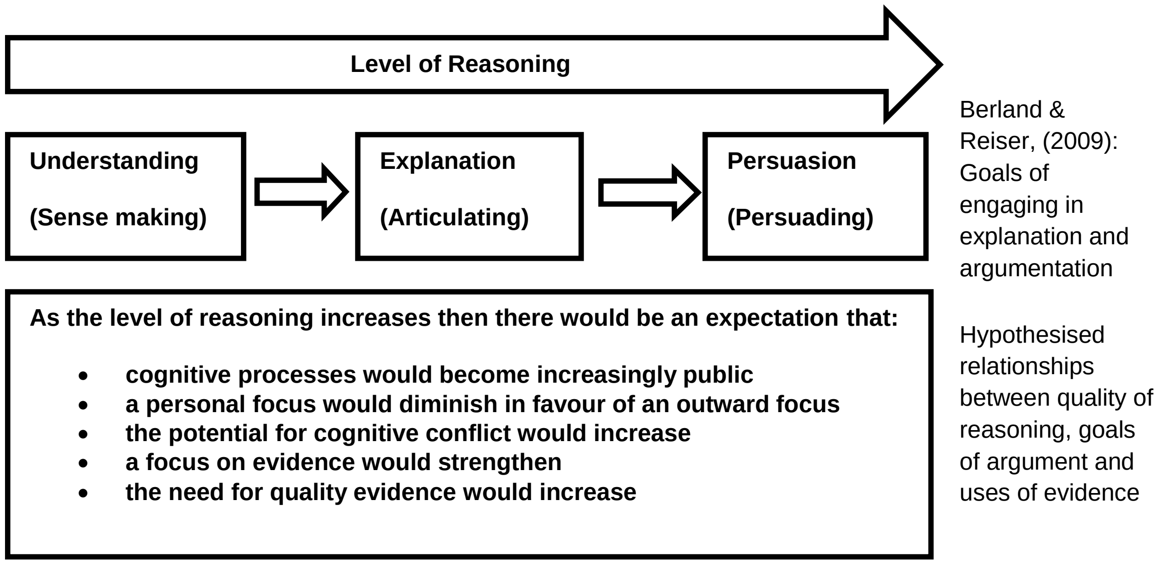


Figure 8.4: Adjusted model of potential interactions between reasoning, goals of argumentation and use of evidence.

(Figure 8.4) rather than necessarily causal, as the original diagram (Figure 8.3) implied.

8.5 Components and Role of Mathematical Knowledge

The proposed model of IBA (Figure 8.1) also incorporates multiple components of *Mathematical Knowledge*. The first of these components to be addressed is prior experiences and influences. Aspects of mathematical knowledge to be discussed include conceptual and procedural knowledge, community practices, the role of discourse, and affect. Again, this should by no means be considered comprehensive.

8.5.1 Conceptual and procedural knowledge

One of the considerable concerns of reform mathematics (Wu, 1997) is the possibility for mathematics and mathematical language to become lost in the quest for a contextual focus and it was this issue that largely provided the impetus for the research here. By implementing argumentation practices in the classroom, the resultant focus on mathematical evidence and reasoning was hoped to deepen the mathematical understanding of the students. There were two primary sources of evidence that suggest students' development of deeper understandings across the units: the Epistemic Criteria rubric (Table 5.7) which was used as a tool for the researcher to identify changes between students' arguments; and the students' discursive contributions which revealed changes in understanding.

The students written arguments were examined and scored against the rubric for Epistemic References across seven categories on several occasions over the course of the research (Table 7.5 provides a summary): *Evidence Collection*, *Foundation for Evidence*, *Evidence Gathering*, *Organisation and Representation of Evidence*, *Evidence Interpretation and Analysis, Anomalies or Contradictions in the Evidence*, and *Reasoning*. Briefly, in each section, an explanation of what the category entails (as developed from open coding, see Section 5.7.2) is provided.

Evidence collection

This category seeks to determine if the evidence the students are collecting actually responds to the question being asked. Essentially, it addresses whether the students could actively address the question with the evidence. The importance of this is the ability of the students to envisage evidence that could be used and the mathematical procedures and knowledge that would be required. In traditional methods of learning, students often are asked to address textbook questions, workbooks, or worksheets where they are either provided with an algorithm, or the solution method is located close by. The nature of the questions asked in ill-structured inquiry is that the students would typically not be cued in such a way (Barbie unit) or might be cued to only a broad area of focus (Pyramids).

Foundation for evidence

It is necessary that the mathematical evidence provided is based on accuracy and fact as distinct from conjecture, supposition, opinion or fallacy (as defined by Toulmin,1984).

Previous research (Muller Mirza et al., 2009) suggested that students may rely on evidence or observations they have to hand if they have no factual evidence. However, the students, in practice, rapidly focussed on the need for obtaining evidence on which to base their conclusion. Several possibilities for the discrepancy suggest themselves. The first is that these students had already taken part in IBL and were aware that evidence was necessary. In the Barbie unit, the students did not initially seek evidence but very quickly shifted to a desire to 'test' to find out. It is more likely that, as previous findings relate predominantly to science, there is a difference between scientific argument and the argument that is being proposed for mathematics. In Science, students often come with alternate conceptions based on their observations of the world around them: the sun moves around the Earth because we 'see' it do so. Thus in Science there is potential for deeply ingrained 'understandings' to exist with students prior to the Inquiry.

Evidence gathering

In epistemic argumentation, it is necessary that evidence is appropriately gathered given the community standards. However, the students need to consider whether providing the method of obtaining the evidence assists the audience to evaluate the evidence. In some instances, it is sufficient that the audience is provided with evidence (for example, a scalene-faced pyramid). In other instances, the method of gathering information may be crucial to its interpretation, for example, aspects of statistical inquiry, such as sample size and sample selection, which are required to enable judgement to be made about validity. In these instances, details of method should be provided and should be considered acceptable within the standards of the community.

Organisation and representation of evidence

Representation of information is a significant aspect of mathematical work (Boulton-Lewis, 1998; Cifarelli, 1998; NCTM, 1991). Users of mathematics may sketch rough maps and diagrams, scribble workings on scrap paper, build models and so on. Students need the opportunity to decide on, and compare and contrast the usefulness of various representations. In both of the units, mathematical representation took on a significant role in working with evidence. In the Barbie unit, the students predominantly worked with their own representations and then engaged in class discussion to contrast those representations and decide upon a dot plot, which was collectively refined to include scaled intervals. This gave the students the opportunity to engage in discussion around different representations and the subsequent design of their own.

In both units, the students developed a greater focus on representations as tools to assist with thinking, planning, checking, and finally to be used as evidence for the audience, as the nature of the audience dictated to some extent the nature of the representations presented in the final argument. Across the course of the study, students came to realise the importance of representations and this developed with regard to their need to interpret the data and convince their audience. Their initial, minimally organised data scores can be compared to their final, carefully crafted dot plots in the Barbie unit (contrast Oliver's early attempt with his final attempt in Figures 6.7 and 6.8). Whereas in the Pyramid unit, students were careful to produce ever-increasingly accurate versions of their pyramids in order to produce quality evidence (Lines 107-110, Section 7.5.1, suggest the preciseness the students were aiming for). Table 7.5 provides a comparison of mean scaled scores for the initial and final responses for each Inquiry. Within each inquiry, there is a clear improvement in the Representation and Organisation of Evidence scores: 1.9 to 4.3 for Barbie and 2.1 to 4.2 for Pyramids. This reflects a within unit improvement, rather than a between unit improvement which may suggest that argumentation practices serve to further students' knowledge of, or development of, useful mathematical representations within the specific field of mathematics with which they are working. Further research as to any transferable features, such as overall accuracy, in students' representations would be interesting.

Evidence interpretation and analysis

In applying mathematics to solve inquiry problems, the findings must be analysed or interpreted in order to draw conclusions, and the conclusions must be applied back to the initial problem. Having the students express a claim, and then ensuring it addresses the research question (and context if appropriate) situates the students to attempt to make sense of their results. An example of the difference is provided in the samples of Oliver's work. His early claim simply consisted of a dot plot with Barbie's score separately identified from the human scores (Figure 6.7). However, his final response (Figure 6.8) fully articulated a claim, stated the evidence while providing an accurate representation, and stated that Barbie is not within the cluster of human scores. In order to be able to make this claim, Oliver interpreted the data and took into consideration the range of scores, the distance of Barbie's score from the range, and what these scores mean in terms of proportion.

Anomalies or contradictions in the evidence

Both Sampson and Clarke (2006) and Zeidler (1997) address the unacceptability of students selecting evidence according to their own viewpoint, or avoiding evidence which is contradictory to a claim. Doing so, and discounting oppositional data, is not an accepted practice within scientific or mathematical inquiry. Scientists and mathematicians seek to explain or at the very least report anomalous or contradictory evidence because: the basis of scientific discovery can often be found in the evidence that didn't behave as was expected; it is ethical; and, because identifying weak points serves as an indicator to others that there may yet be something more to explore and explain. As anomalous results have the potential to be important in scientific and mathematical research, we should attempt to explore such results and ensure that they are anomalous and not simply erroneous. This was a focus during the Barbie unit, and was assessed in the final task when students were purposely provided with a data set that had extreme scores. While the majority of students were willing to identify these scores as anomalous ('mistakes', 'errors', or 'outliers': see Figure 6.7 for an example), they did not attempt to check their accuracy to ensure this was the case, despite being prompted to do so. In hindsight, this should have been further explored with students.

Reasoning

The reasoning provides the link from the evidence to the claim by explicating the mathematical rules, laws, and conventions that the students have relied upon: justifying their claim based on the evidence. Essentially, this is the decontextualized mathematics or mathematics in its 'purer' form. If the example of the Barbie unit is used, one student made a claim that "Barbie dus not have human proportions for this meserment. Barbie is an outlier and is not in the cluster. … The cluster is from 1.2 to 1.6." (Oliver, Figure 6.7).The reasoning is not explicit but implied – the cluster is where the range of normal sample scores lies and the value we are looking at does not lie within that cluster, therefore that value probably does not come from the population from which the sample was drawn. This expresses the students' underlying and developing informal inferential reasoning.

By the Pyramid unit, the students were being taught about the nature of reasoning explicitly, and were asked to express the mathematical understandings they had drawn upon to make their argument. Hence, students made statements like Connor's (Figure 7.11), where he explains the properties of a pyramid and of scalene triangles. Table 7.5 includes the mean scaled scores for reasoning across and between units, and the increases in the 'Pyramid' unit likely reflect reasoning being introduced explicitly in the second unit.

8.5.2 Community practices

A second component of Mathematical Knowledge reflects the nature of the learning environment. One of the recurring phrases during the discussion on conceptual and procedural knowledge is 'within the community' or 'within the standards of the community'. The phrase 'given community standards' has been repeatedly and purposely built into the Epistemic References rubric; the reason being that these students are not practicing mathematicians but rather mathematical apprentices (Collins et al., 1989) and, as they develop, their mathematical practices need to increasingly approximate the authentic practices of mathematicians. Thus, the students' practices need to be acceptable for the age and stage they are at. For the Barbie conclusions it was pleasing that students could explain that Barbie was too far removed from the sample to be considered part of the sample, not disappointing that they couldn't manage a t-test. It is this approximation of the mature discipline that is the goal in developing a community of mathematics apprentices, rather than a

> rose-coloured notion of students entering the community of practice of adult academic disciples …awareness of the deep principles of academic disciplines should enable us to design intellectual practices for the young that are stepping stones to mature understanding or at least are not glaringly inconsistent with the end goal. (A. L. Brown & Campione, 1996, p. 306)

By taking part in the community practice of mathematics, students importantly develop understanding of what is important and valued by the mathematical community. When these students commenced the Barbie unit, only two students selected data as the essential feature in convincing of the accuracy of their claim (Table 6.6: Data Interpretation and Analysis and Data Representation). By the conclusion of the Pyramid unit, all students were providing objective data.

8.5.3 Discourse

Discourse is an important component of building mathematical knowledge. Through engaging in discussion, students have the opportunity to express ideas, have them challenged and thus develop robust reasoning. However, they also have opportunities to use the language and vocabulary of mathematics and this helps to develop their understanding of concepts and terms within a context and thus learn the correct usage and meaning. In turn, this gives students the opportunity to express themselves precisely. Towards the end of the Pyramid unit, the students were quite comfortable making precise statements such as: “Our group has made two pyramids. One is a triangular-base (pyramid) with three scalene faces and our other one is a square base pyramid with two scalene triangles and two isosceles triangles.” (Sadie, Figure 7.8). At the commencement of the unit, the students were referring constantly to their mathematics books to remind themselves of what scalene and isosceles meant.

Mathematics is an area of formal schooling in which the literacy demands are quite high, with students constantly being exposed to new, often abstract and challenging terms, many of which have little or different meaning outside of mathematics. The opportunity to use these terms and develop contextualised meaning is vital to students’ engagement in mathematical practice (Lee & Herner-Patnode, 2007; Rubenstein & Thompson, 2002).

8.5.4 Affect

Affect, like discourse, was not a specific focus of this research but warrants inclusion in this model and is flagged as an area of future research. Mathematics is a subject area that has been characterised by poor student disposition (McPhan et al., 2008). Early research reported elsewhere (Fielding-Wells & Makar, 2008a, 2008b) suggests that IBL has potential to curb and even reverse aspects of disengagement in mathematics among elementary and middle school aged children. The nature of the students repeated attempts to move forward, while meeting resistance, and negotiating ideas, and then moving forward again has potential impact on students’ mathematical resilience: another area that would warrant further research.

The identified components of Mathematical Knowledge have been addressed in this section. Conceptual and Procedural knowledge were given by far the greatest coverage as these were a significant focus of the research and essential to students’ learning of mathematics. However, the twin underpinning principles of Fostering Communities of Learners and Knowledge Building (Scardamalia & Bereiter, 2013) are reflected in the Community Practices component, making this an important consideration. Discourse and affect were paid passing acknowledgement, not because they lack importance, but because they were not a focus of this research. Certainly further research in both areas within IBA is warranted.

The identified components of Mathematical Knowledge have been addressed in this section, with conceptual and procedural understandings taking a privileged position due to the importance of addressing these in student learning. Community practices are important to consider, as a shift towards a Knowledge Building approach appears ideal in developing beneficial attitudes (affect) towards mathematics. Discourse is critical, and again, while it was not a major focus, there is much research to indicate the critical nature of mathematical discourse in the classroom (Lee & Herner-Patnode, 2007; Rubenstein & Thompson, 2002), and IBA offers affordances for developing mathematical discourse. The role of Mathematics Knowledge within the practice of IBA will be addressed in the following section.

8.5.5 The role of mathematical knowledge

Mathematical Knowledge takes on several roles within the IBA framework, providing the tools with which to address questions, obtain evidence, organise, analyse and interpret the evidence in order to make a claim. The conventions and rules of mathematics provide the reasoning to justify the evidence-to-claim link.

The nature of the Inquiry-Based Argument as implemented here is such that the question must be addressed by students using mathematically-derived evidence. Thus the students are situated in such a way as to need to envisage the mathematical evidence, and mathematical concepts and procedures that would be useful in addressing the question. In the Barbie unit, students could envisage what needed to be done – finding a means of comparing a Barbie doll to a human – but the difficulty arose in that the students did not know how to do that. This quandary gave students their first insight into a mathematical need for proportional reasoning. It was then that the challenge as to whether one human would be representative of all humans necessitated a more formal means of calculating proportion to enable the comparison of many. As the students did not have these ‘tools’ in their repertoire, the need provided an opportunity for the more formalised introduction of proportion (Lines 35-38, Section 6.3.3) largely through direct teaching and discussion. Thus the need for evidence drove the need for mathematics in the context.

Once the students had the proportions, their previous use of dot plots and tallies enabled them to organise the data; however, they were not familiar with the concepts of distribution. Again, the need for evidence drove a requirement for deeper mathematical understanding, this time developed through challenge and discussion rather than direct teaching. Statistics provided the means for estimating the population and inferring whether Barbie might sit within that population. The concept of sample as representative of the population was demonstrated by a number of students; however, around half of the class continued to treat the sample data as if it were population data (see Oliver’s final response in Figure 6.7 in which he made no attempt to extend the sample cluster to incorporate even the likely score of 1.0). The students applied their newly acquired proportional understanding to address the issue of how much spread would be likely. Many of these concepts were informal and would need to be addressed again over time. Despite this, the understandings demonstrated were higher than anticipated and above curriculum expectations, even amongst the students that had previously experienced difficulty with mathematics.

With the Pyramid unit, the absence of an externalised context potentially reduced or removed external distractors. Because of the wording of the question, students had little difficulty envisaging what was needed in terms of evidence, as the question itself pointed to the mathematics required. However, this also removed the necessity of students having to consider what mathematics might be applied to the problem, thus losing some of the authenticity of real mathematical practice.

The three domains identified as a part of the IBA model have been discussed separately, and largely broken into their individual components. However, the paragraph above shows the importance of the intertwining between the domains and the components of the domains. Working with a decontextualised Inquiry question had implications for the students’ engagement with the argument structure and the way in which they applied mathematical knowledge. In the section following, some of the key interactions between the domains will be addressed.

8.6 Interactions between Mathematical, Context, and Argumentation Knowledge

One of the more important findings suggested by this study relates to the nature of the interactions between Context Knowledge, Argumentation Knowledge and Mathematical Knowledge (Figure 8.1) with regards to how each of these could support and scaffold the others.

Context Knowledge – Argumentation Knowledge

Context Knowledge has the potential to provoke a need and provide support for the specific teaching of argumentation practice. The argument itself is made up of three components - claim, evidence and reasoning - and the context has potential to provide scaffolding for each. As the claim should be drawn from the evidence, evidence is the first aspect of the argument the students will usually engage with.

In the Barbie unit, students first drew on their prior, contextualised experiences with Barbie dolls, and on their experiential knowledge of humans, in order to provide non-mathematical evidence to assert that Barbie could not be human. This suggested that the students' assertions were based on the evidence they had available, contrasting somewhat with previous observations that students' claims are typically impulsive and tend not to be supported with evidence (Fielding-Wells, 2010; Muller Mirza et al., 2009). Instead, students drew their evidence from contextually-bound experience. This has potential ramifications in that it implies students must see a need for supporting their assertions; however, they may focus on whatever evidence is readily available and intuitive, rather than seeking verifiable and defensible evidence. It became quickly apparent that the students recognised the strength of a mathematical/statistical response as, once a focus on obtaining mathematically verifiable evidence commenced, there was no further attempt from students to revert to the use of experiential knowledge. This suggests the students recognised that the mathematically obtained evidence offered stronger support than their experiential and observational evidence. One role for the teacher in IBA may be provoking and maintaining a need for more defensible evidence, as was the case in this instance.

By contrast, the Pyramid unit, which was largely decontextualized, did not require such impetus from the teacher, with students immediately setting out to gather mathematical evidence. Three potential reasons are suggested:

1. in the absence of contextualised prior experience, the students had no existing or observational knowledge to draw on;
2. discussion around the need for evidence prior to engagement with the question had focussed the students on evidence; and/or
3. the students had now engaged with these problems sufficiently to see the need (or expectation) of mathematical evidence.

Context clearly had an impact on the development of students' understanding of argumentation structure. In the Pyramid unit, the familiarity with the mathematics, and lack of a confounding context, enabled students to develop an appreciation for the structure of the argument quite easily. However, it could be argued that the mathematical content addressed in the Barbie unit was more challenging and the students were able to use the context to scaffold their appreciation for proportion.

Context Knowledge – Mathematics Knowledge

Context Knowledge also provoked a need for the mathematics and provided a support for the specific teaching of mathematics. From the early stages of the first unit, students were observed to use the context to develop and support initial understandings around the mathematics being addressed. This commenced informally with students wrestling with their emerging ideas of how Barbie and a human could be made the same size to enable comparison. Students then built on their emerging understandings to identify their need for proportion as a mathematical concept (Lines 35-38, Section 6.3.3), and how to represent measures of proportion mathematically. Finally, context was used by the students as a means of determining whether calculations were likely to be correct or not and thus to check the accuracy of the recently learnt formula with which they were working: relying on their contextual knowledge to scaffold the mathematical concepts. For example, when students were developing their understandings of proportion, the context enabled them to build and comprehend proportional representations (Line 34, Section 6.3.3 illustrates Gemma's group's first attempt at conceptualising comparison through proportion).

Later, the context acted to support developing informal statistical reasoning: enabling students to identify problems with comparing Barbie to a single value of the population (one adult female) as distinct from a sample of the population. The teacher was able to lead students to informal consideration of sample sizes and to the need for sampling. This in itself is an important awareness for students to have, as rarely at this age are students aware that there is a difference between population data and sample data, yet it is the need to sample, and to draw inferences from samples, that is the site of statistical reasoning (Pfannkuch, 2006). By enabling the students to have a context to pin statistical reasoning upon, they were able to make decisions with reference to the context, as would ‘real’ statisticians.

In this sense the context also enabled discussions of the limitations of sampling and allowed conjecturing around ‘what ifs’: what if larger samples, or samples from different cultural cross-sections were to be obtained. Unfortunately, students usually get very little opportunity to reason and conjecture in this way in mathematics and the context provided a means to do so. This focus on reasoning and conjecture is again one area which the NCTM (1991) has been calling on for more than two decades and which yet has only recently been embedded in the Australian Curriculum (ACARA, 2014a). Further research is needed into the development of these reasoning skills and whether those skills transfer across situations.

By contrast, the second unit, in the absence of a situated context, was lacking in two main aspects: it did not offer the opportunities to scaffold mathematical knowledge, nor did it highlight the links between real-life mathematical problems and classroom mathematics. This aligns with J.S. Brown et al.’s (1989) assertion that contextual features are necessary for authentic engagement, and that neglecting context when creating classroom tasks can result in a lack of authentic supports. In the pyramid unit there was no representation beyond a familiar image of a square-based pyramid. When the students struggled, there was no referent point beyond the pyramids they were struggling to create, which they could use to check responses. Hence, when students made blatant errors, for example incorrectly measuring angles, there was no cause for the students to question results.

Reflecting on the two units implemented here, limited though the examples are, would suggest that there may be benefits to having students experience both practically contextualised and non-contextualised inquiries while maintaining the argumentation focus. The contextualised focus provided rich opportunities for students to become aware of the usefulness of mathematics in addressing real problems, and to have the additional support that a context can offer in terms of depth of knowledge being built. However, mathematically focussed units may have benefits in terms of students being able to see a purposefulness and appeal of mathematics within itself.

> Unfortunately, students are too often asked to use the tools of a discipline without being able to adopt its culture. To learn to use tools as practitioners use them, a student, like an apprentice, must enter the community and its culture. Thus, in a significant way, learning is, we believe, a process of enculturation. (J. S. Brown et al., 1989, p. 33)

The question may be, which culture are we inculcating students into, and are there two – the practices of the practitioners and of the 'just plain folk' (J. S. Brown et al., 1989) or do we count three, and include the practices of the pure mathematicians?

Argumentation Knowledge - Mathematics Knowledge

Finally, Argumentation Knowledge served to support students' developing Mathematical Knowledge, with the students' focus on evidence essential to the search for mathematics they could use. Throughout the inquiries, it was the need for evidence that drove the students to envisage and identify the mathematics required and the process of challenging each other's evidence, and anticipating challenges from others, that drove the need for quality mathematical evidence.

However, there were also opportunities for the Mathematical Knowledge to build understanding of Argumentation Knowledge. In the Pyramid unit, Mathematical Knowledge was a strength in terms of enabling students to easily envisage evidence needed, and this assisted the students to explore Argumentation Knowledge in much greater depth. In terms of Mathematical Knowledge, the students were largely familiar with the initial geometrical understandings required. As a result, the teacher was able to use their existing knowledge to build understanding of Argumentation Knowledge and adding 'reasoning' and incorporating the structure of an argument into the Evidence Model (Figure 6.1 & 7.1), thereby creating an Argument Model for the students' use (Figure 7.9). As the students had no difficulty envisaging the evidence they would require, it was also an opportune time for the teacher to introduce the issue of quality of evidence. Essentially, the developed Mathematical Knowledge served to scaffold a deeper understanding of Argumentation Knowledge by enabling class discussion on aspects of evidence. However, it was these discussions about evidence quality that conversely resulted in the students rejecting their initial malformed pyramids as not accurate enough. Thus Mathematical Knowledge drove the need for Argumentation Knowledge which drove the need for Mathematical Knowledge. In this respect, we can see that even in the absence of Context Knowledge, Mathematical Knowledge and Argumentation Knowledge can serve to support and raise the level of each other.

A significant difference between the two units was the presence of a context external to mathematics. In the Pyramid Unit, Context Knowledge was virtually absent although it could be argued that the context itself was within the field of geometry. This has several implications for IBA. Firstly, by removing an external context, there was limited opportunity to connect this unit to real-life mathematics or mathematics situated within other areas of the curriculum: a loss of potential for building real-life or cross-curricular connections as recommended by the NCTM (1991). However, in mathematics there is a role for pure mathematics as distinct from applied. Hence there is an argument that having students act as ‘pure’ mathematicians, seeking to answer puzzling questions about mathematics itself is invaluable: as is the opportunity for students to see that they can ‘wonder’ about mathematics and apply their knowledge and skills to address interesting aspects of mathematics itself. In turn, this suggests to students that knowledge about mathematics may not be absolute and fixed but in fact open to authentic knowledge-building. Conversely, removing the non-mathematical context removes one source of opportunity for scaffolding the Mathematical Knowledge and the Argumentation Knowledge. Potentially then, there is a role for both forms of IBA: contextualised and non-contextualised.

8.7 Summary

In order to address the first research question, a model for Inquiry-Based Argumentation was presented which proposes three knowledge domains: Mathematical Knowledge, Argumentation Knowledge, and Context Knowledge. While the first two components are necessarily present in IBA, the need for the inclusion of Context Knowledge in every IBA unit is questionable: there may be benefits to engaging students in both contextualised and decontextualised mathematical inquiries. Many of these benefits would stem from potential opportunities to use the context to scaffold the mathematical concepts and the knowledge of argumentation product and process. However, the context also has the potential to be a distractor from the mathematics. While we saw examples of this in the initial stages of the Barbie unit, it was not difficult to focus the students on the need for mathematical evidence. Certainly, further research is needed, not only into the domains identified here, but into the components of each domain and the way in which the contribution of each component has been visualised here. Additional research into the seemingly complex nature of the interaction between domains would be a priority in developing further understanding of IBA. Hence, this model is proposed as a starting point for future examination, critique, and development, in an attempt to refine and define the essence of Inquiry-Based Argumentation in mathematics.

The remainder of this chapter will address the focus of the second research question by proposing Signature Elements of IBA; that is, those elements that would be considered essential to the practice of IBA. These have been addressed in two sections: those that are considered crucial for students to engage with, and those that are more advanced and perhaps ‘optional’ in the early stages, but which are critical elements in the longer term for extending the practice of IBA.

8.8 Signature Elements of Inquiry-Based Argumentation in Primary Mathematics

The second research question addressed in this dissertation was, “What Signature Elements of Inquiry-Based Argumentation can serve to guide children’s mathematical argumentation?”. Thus, the purpose of this section is to establish the elements which appear to be characteristic of Inquiry-Based Argument (IBA); that is, the Signature Elements. The purpose is to establish criteria by which to categorise or identify an activity as IBA. Students engaged in learning at the school level have a wide variation in developmental levels, and it would be inappropriate to suggest that 7-year-olds with no experience of argumentation or argumentation practices could produce an argument at the level of a 13-year-old who has been repeatedly engaged in constructing mathematical arguments. Drawing on the wider research background, and based on the results presented in the previous chapters, this section will suggest elements which would be essential to IBA at both initial and advanced levels. The idea of essential and advanced elements reflects far less on the students' ages than on their exposure to IBA, their discursive development, and the culture of the classroom.

8.8.1 Essential elements of argumentation-based inquiry in mathematics

At the very simplest level, mathematical argumentation can be characterised by student engagement in addressing a *purposeful inquiry question*, the *advancing of evidence which is used to form a claim,* and the *justifying of the evidenced claim* through *epistemically acceptable reasoning* in *context.* Each of these elements is an integral and essential aspect of the Inquiry-Based Argument unit. While the elements are presented here sequentially, in practice the teacher draws attention to different components and the relationships between different components, as required. Each of these elements will be addressed in turn.

Addresses a purposeful inquiry question

In order to present an argument, students require a question they can address. Any question posed should essentially be interesting, novel, worthwhile, personally relevant and/or clearly 'real' (Fielding-Wells & Makar, 2008b): it should not be contrived or have little value to the students.

The Inquiry question may be very broad and require refining, "*Is Barbie a Human*?", quite specific, "*Does Barbie have human proportions*?" or very focussed, "*Does Barbie have human proportions for the ratio height: arm length*?". Each of these question types serves a different purpose. The first question is exceptionally broad and offers students the opportunity to narrow and refine the question: to determine what is meant by 'human' and use this to identify and engage with Mathematical Knowledge. In this way students have the opportunity to determine the conceptual and procedural knowledge that can be applied. Further, conversations that students engage in, for example disagreeing about the length of Barbie's neck (Lines 22-28, Section 6.3.2), appear to act as a 'hook' to engage the students in the inquiry (Allmond et al., 2010), which in turn reflects the *affect* component of Mathematical Knowledge. By overly refining a question, many of these opportunities may be lost.

Inquiry questions need not be posed by the teacher. In the instance of the pyramids, a student's question was adopted after it was posed spontaneously in class. Students are capable of formulating their own inquiry questions even from a young age, although research indicates there is a need to teach students how to pose their own questions with a focus on what makes a 'good' question (Allmond & Makar, 2010). While this may be time consuming it does more closely match authentic practices and teaches students an important skill – how to mathematise a problem so that it can be addressed.

The word purposeful has been added to the element *addresses a purposeful inquiry question*. For clarity, a purposeful question is one that seeks to address a genuine problem. The authenticity of purpose is an important feature. Sandoval and Millwood (2007) argue that often when students are provided a question, the teacher already has a known answer. Because of this, even if the question is open-ended, students may not engage purposefully as they have no real need to persuade their audience (the teacher) of the answer or a method. In the units presented here, there was a tension between creating an audience (the class) and awareness that the audience was familiar with the detail, and may have less need of details or persuasion. The teacher made constant reference to the idea of convincing 'other' people (these others were never actually articulated but rather presented as an abstract idea: Lines 51-68, Section 6.4.2, Third Iteration, illustrates the teacher's use of the 'other' audience). The authenticity could be improved if a genuine audience outside of the classroom could be identified. The influence of a genuine audience on the argument is certainly worth further research.

An argument could also be posed that the question does not need to be purposeful, that any mathematically researchable question would suffice. While this is true, the purpose goes a long way to showing students that mathematics has usefulness and that there is benefit to its study. The purpose also serves to engage students as they solve a problem they can see has merit. Neither of the questions posed in this research meet these criteria as well as the researcher would like; the first felt forced into an existing context, and the second had no context outside of mathematics. However, these are the constraints of working within school systems and the constraints are genuine; as such they may be broadly experienced by teachers.

<u>Advances evidence to enable the forming of a claim</u>

In persuasive (expository) texts as taught at the school level, students are encouraged to take a position and then defend it using persuasive devices in order to bring the reader to their point of view. These devices may include the active and intentional use of such methods as those Toulmin et al. (1984) lists as fallacious: *appeal to the people* and *appeal by force* being two methods in common usage in advertising or debating. However, in scientific (and mathematics) practices, we seek evidence first and then seek to make sense of it; to draw conclusions based on the evidence we have, both supportive and contradictory (Sampson & Clarke, 2006).

In order to advance evidence, students must be able to envisage the evidence they could use to address the problem, plan to obtain that evidence, organise or represent the evidence and then interpret and analyse the evidence they have. Students may then make a claim, provide the evidence, and explain their reasoning that links the two. However, in advancing the evidence to an audience, there is typically a requirement for a representation of the evidence to be provided; as the representation itself becomes the tool for thinking, interpreting, reasoning and communicating (Greeno & Hall, 1997). When working with Barbie, with the purpose of sense-making (Berland & Reiser, 2009), the students initially produced largely unorganised results (Section 6.4.2). As they began to share information with others, and considered the needs of an audience, students' representations became increasingly more ordered and more purposefully organised. Gains in students' mean scaled scores were notable across each unit (Table 7.5) as they shifted from internalised understanding to a focus on explaining and reasoning to an audience (for examples, see Lines 60-69, Section 6.4.2; Lines 81-83, Section 7.2.4). However, there are also instances where the students clearly used their representations to aid their own understanding and then explanation to others (Lines 115-118, Section 6.5).

The final step in this element is the forming and articulation of a claim which accounts for all the available evidence: including any evidence which may be contradictory to the overall conclusion (Sampson & Clarke, 2006). In Epistemic Argumentation, the goal is to reach a conclusion that best explains the evidence at a point in time and this includes acknowledgment, and where possible, explanation of conflicting evidence (Siegel & Biro, 1997). In Knowledge Building communities, ideas are considered improvable (Scardamalia

& Bereiter, 2006) and the goals of Epistemic Argumentation align well with a philosophy of knowledge improvement.

In both units there was a focus on improvability and the sense of the conclusion being the best available at that time. For example, in the Barbie Unit, students acknowledged that there was a possibility of obtaining more data and that this may change their responses. In the Pyramid unit (Lines 117-130, Section 7.7.2), the class openly engaged in a discussion about modifying claims to make them more accurate as distinct from 'stronger' through the inclusion of additional evidence. Once the evidence has been advanced, students need to make a claim, articulate it clearly and then justify the claim through provision of the evidence and reasoning to demonstrate how the claim derives from the evidence.

Justifies the claim through epistemically acceptable reasoning

The final essential element is the use of reasoning to justify the making of the claim: reasoning which is based on evidence. There is potential for the connection between evidence and claim to be omitted, largely because the connection is either thought to be implicitly understood, or is left unaddressed unless challenged. However, this does not meet the purposes of IBA in mathematics as the reasoning is the site of the actual mathematical understandings, connections, proofs, or concepts. In science education, Zembal-Saul et al. (2013, p. 25) state that reasoning should include the science idea or concept that is the focus. To omit this does not enable affirming of a claim's validity (Zeidler, 1997). In scientific and mathematical argument, that connection, or reasoning, needs to be made explicit in order to open it to critique, challenge and ultimately, to develop the most robust conclusion with the evidence to hand (Sampson & Clarke, 2006). An example of a deconstructed argument is provided for illustration (Figure 8.5). This was not an actual student argument but rather the gist of many presented and recorded in mathematical terms. As can be seen, the reasoning presented would be consistent with the knowledge students are expected to have in line with state curriculum requirements (Appendix B), with only the sum of internal angles, and the ability to measure those angles, considered ahead of the current year level. Likewise, the reasoning presented is consistent with widely established disciplinary understandings. Students would not, at a future point in time, learn something contradictory to what they have stated in their reasoning.

While the suggestion here is that the signature components for argumentation should include claim-evidence-reasoning (McNeill & Krajcik, 2008, 2011; McNeill, Lizotte, Krajcik, & Marx, 2006; McNeill & Martin, 2011; Zembal-Saul et al., 2013), it is only essential that the teacher be able to recognise these components, particularly in younger students, and that these components may be elicited, for example, verbally, pictorially, diagrammatically, or concretely. This again reflects the importance of considering what is appropriate for a specific learning community. However, to have the students accustomed to providing evidence for and justifying their responses even at an earlier age would likely position the students for more formal learning and reasoning at a later time.

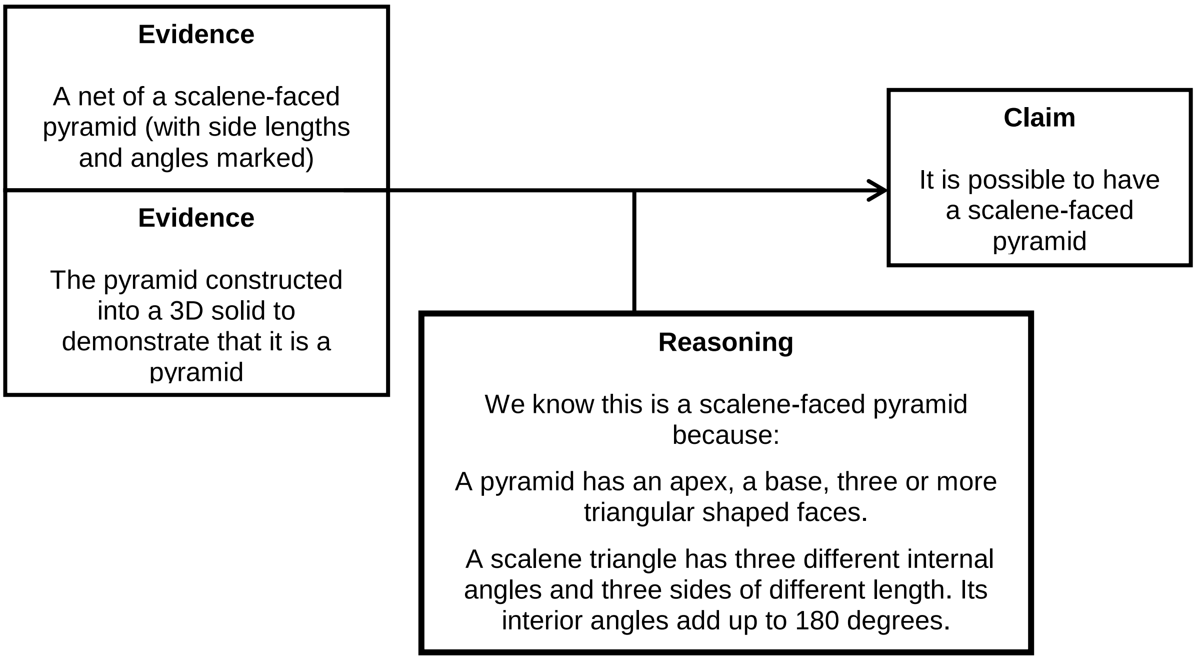


Figure 8.5: Sample of mathematically explicit reasoning

Reflects the context

The final element is the necessity of the claim, evidence and reasoning, to reflect the question context or purpose. In mathematically contextualised units, this may well be incorporated into the claim; for example “A pyramid can have a face that is scalene” (Figure 7.10). The evidence provided demonstrates a pyramid with a scalene face and the reasoning demonstrates that the face is scalene and that the solid is in fact a pyramid.

However, in units which are contextualised outside of mathematics, such as the Barbie unit, the students need to make an overall assessment of what their response means to the context. While the claim would reflect the context and the reasoning would require a mathematical basis, the evidence may be constrained or guided by the context. This could potentially influence the evidence at several stages, for example:

- Envisaging (How many people should be measured? Should we measure only females? Should they be adult or can they be any age? What measures should we take?).
- Interpretation (What does the evidence mean in light of the context? How can anomalies be interpreted in light of the context?)

In the Barbie unit, there are multiple occasions when the context impacted on the Inquiry itself. However, perhaps the most indicative was reflected in students' desire to limit what was possible for the population by considering the context of human proportion. This was represented in Geneva's quandary about how extreme she could make her range of normal proportions for a human, given what it would mean in terms of human appearance (Lines 136-152, Section 6.5).

While there has been research to suggest that disengaging or overly sophisticated contexts, or contexts likely to provoke very strong feelings in students, can all be counter-productive to engaging with argumentation (Douaire, in Muller Mirza et al., 2009), the contexts provided here were not of such a nature. Contexts which were found in previous work to engage students more highly in IBL included: contexts of high interest; those students could relate to in terms of real-life or personal experience; those perceived to be of high value; those which enabled students to experience enjoyment; or, those that were novel or challenging (Fielding-Wells & Makar, 2008b). The results from this research do not contest those findings; however, they likewise were not broad enough to offer significant additional support. What the research conducted here does suggest, and this is addressed in the section on Context Knowledge (Section 8.3), is that context has an important role in IBA (and IBL) and warrants further research.

8.8.2 Advanced components of argumentation-based inquiry in mathematics

The previous section addressed those components of mathematical argumentation that are considered essential to the undertaking or Argumentation as a mathematical pedagogy. However, the development of the students in the research project reported here would suggest that, as students become experienced with argumentative structures and with the conventions and culture of argumentation processes, there are further skills that they would benefit from developing. These are not add-on extras but rather advanced components thought to support deeper reasoning and lead towards a classroom that is more aligned to Knowledge Building. Thus, at a more advanced level, students would *critically examine their own conclusions and those of others*, *define the parameters of their arguments*, and *consider audience*.

Critically examine their own conclusions and those of others

To advance Knowledge Building within the classroom, students need to become accustomed to the practice of *Knowledge Building Discourse*, a view of knowledge as "refined and transformed through the discursive practices of the community – practices that have the advancement of knowledge as their goal" (Scardamalia & Bereiter, 2010, p. 10). These discursive practices include the need to critically examine the conclusions (claim-evidence-reasoning) of others and themselves in order to advance Community Knowledge (Scardamalia, 2002). For students to critique the work of classmates required something of a shift in the culture of the classroom, and this would be anticipated to be the case in most classrooms, as students were/are not accustomed to having their responses challenged by their classmates. Some insight into this aspect came early in the Barbie unit.

From very early in this unit, students were able to see a need for evidence, and could explain the purpose of the evidence as being able to answer the question, to support the conclusions, and to provide detail (for example, see Line 33, Section 6.3.4). While the students suggested it was important to ensure people could 'believe' them, they did not immediately see the role of evidence in helping to convince. When students were asked to focus on convincing, and then self-assess whether they thought their conclusion was convincing, there was a limited focus on evidence in their responses. Rather, some students even suggested that how much effort they had put into compiling their responses should influence the confidence of others (Table 6.5). If students felt a personal attachment to their work, including a sense of it being a result of some effort, then it is unsurprising that having their work, and by extension themselves, 'criticised' by others would be difficult and that they would feel uncomfortable reciprocating. However, the very act of critiquing each other's work helped to overcome this view. Once the students had opportunities to critique the work of others, and then compile a list of attributes that were effective at convincing, they began to focus more strongly on aspects of evidence including: inclusion of evidence, clarity of evidence, and reasoning (Tables 6.7 and 6.8). Essentially, students needed to *be* an audience to effectively consider what assisted an audience. This also enabled them to see the perspective of others and potentially decentre from their work. To maintain this focus, the teacher focussed the students regularly on the idea of 'What would convince others?' to have them reflect more critically on conclusions and any inherent weaknesses in those conclusions.

Attached to the idea of critical examination, is the need to consider and anticipate potential criticisms that could be addressed and then respond appropriately. This may mean gathering more evidence, providing a representation in a clearer way, amending conclusions, or preparing a defence. While students were more than capable of baseless arguing with people who disagreed with them, the first instance of a student defending their viewpoint through the use of reasoning came with Geneva (Lines 136-152, Section 6.5) during her presentation of a range of human female proportions. While Geneva was willing to express uncertainty over how wide she should make the human range, she argued for the likelihood of greater variation in the population and wished to extend her range of 'normal human proportion' quite significantly. When challenged on this, she was prepared to argue that maybe her extreme score wasn't possible, but that it could be a person with exceptionally short legs or who had been injured (and experienced an amputation). It was also notable that Geneva defended herself against a teacher's challenge, which, in the Australian culture of schooling, is still fairly unusual and would typically be considered inappropriate for a well-behaved student like Geneva.

The Pyramid unit was undertaken when the students were in their following year of schooling (9 year olds turning 10 during the year) with the students now eight months older. From the early stages in the Pyramid unit, the students were openly challenging each other's ideas, both in groups and whole class, without needing the teacher modelling (lines 111-116, Section 7.5.1). Of perhaps greater interest is that the students being challenged were accepting of the challenges and feedback as a means of improving their work rather than as unwanted criticism. It is not possible to determine the factors that contributed to such a shift in the students' communication practices; however, several suggested factors would be age (maturity), experience of the previous inquiry, a stronger group focus as distinct from the individual focus of the Barbie unit, and/or increased familiarity with the underlying mathematics. What was seen in response was that the students began to anticipate challenges to their conclusions and work to build in information that might address these challenges before being asked. This suggested both an increased awareness of audience and a greater degree of comfort with the idea of having ideas critiqued, and critiquing those of others, for the purpose of building stronger knowledge.

Define the parameters of their arguments

Inquiry problems are by design ambiguous and ill-structured (Anderson, 2002) and the nature of such questions is such that they require refining of and negotiating to a point where they are researchable (Allmond & Makar, 2010). The addressing of appropriate parameters, such as refining the Barbie doll's general 'human-ness' to specific measures of proportion, becomes necessary to enable the students to investigate the question. Parameters can be established through the context, or by negotiation of the problem and the problem purpose. However, such parameters should be acknowledged by students when putting forward their claim. One method for doing so is through the use of delineating qualifiers (discussed in Section 8.4.1), to limit the conditions that the claim applies to.

While McNeill's CER model (McNeill & Krajcik, 2011; McNeill & Martin, 2011; Zembal-Saul et al., 2013) does not incorporate qualifiers explicitly, this would not preclude them from being built into the model through use of a qualified claim (McNeill & Martin, 2011). It would seem that any model of mathematical argumentation, based on ill-structured inquiry, must necessarily incorporate some form of articulation of parameters at the more advanced level.

Consider audience

> Audience is complex. Understanding the nature and impact of audience plays a central role in the vast literature on rhetoric and argumentation. Whether written or oral, arguments are situated in a rhetorical space that is constructed by both the audience and the speaker or writer. (Berland & Forte, 2010, p. 428)

The term ‘audience’ can take on many meanings; however, in the sense of this dissertation, it is defined to mean anyone that the interlocutor engages with in dialogue. In applying audience to Berland and Reiser’s model (2009), the nature and influence of audience change with the shift from the tacit level of sense-making, through explanation, and finally to persuasion. In the Barbie Unit, there was a deliberate movement of students through the phases in order to ‘test’ Berland and Reiser’s model (Figure 4.4, and discussed in depth in Section 8.4.4). The hypothesised relationship was that as students’ focus moved to explanation and persuasion, the existence of an audience that had to be first informed and secondly persuaded would, among other aspects, result in a deeper focus on evidence and evidence quality. While this was apparent in the Barbie unit (refer to Section 8.4.4), as students were progressively focussed on an external audience, such a focus was not required for the Pyramid Unit. In this unit, the students immediately focussed on evidence for the purpose of an audience as soon as the role of evidence was queried (Lines 15-21, Section 7.2.2).

Essentially, in the Barbie unit, it was the responses of the ‘other’ (the audience) and the student’s own experiences of being an audience (when critiquing each other’s’ work in the second iteration of Section 6.4.1), that served to cause the students to modify, construct and reconstruct understandings, and provide opportunities for improved clarity and the development of shared understandings. However, it was not only the responses of the audience that changed the nature of the discourse, but also the awareness of the presence of an audience (Berland & Forte, 2010) as was seen in the Pyramid Unit.

Audience has been referred to repeatedly through the Discussion and it is clear that audience has the potential to powerfully influence the argumentation structure and process. This is an area that needs significant research, especially research into what makes for an effective audience in IBA. For example, would an authentic external audience be more influential than simply having the class as an audience? There would be an expectation that students may be more careful about articulating claim-evidence-reasoning to an audience unfamiliar with the question and process; however, this is only conjecture.

8.9 Summary

In this chapter there has been an attempt to address the research questions which drove this study. First, suggestions were made as to key components of a model of Inquiry-Based Argument, as implemented in a primary (elementary) mathematics setting. Essentially, these components were clustered into three interacting domains of Mathematical (or Content) Knowledge, Argumentation Knowledge, and Context Knowledge. Some identified interactions between these domains were also discussed as, while these interactions were not anticipated, there appears to be potential for components of the model to be used to scaffold understanding in other components or domains. There would be benefit to having these domains, and the nature of these interactions, more extensively researched, particularly with a view to the role of context for scaffolding the development of deep mathematical knowledge. This is especially the case as inclusion of an external context does not appear necessary, but may be highly beneficial for developing mathematical connections.

The second research question addressed the identification of Signature Elements of Inquiry-Based Argument that might serve to guide the implementation of children's mathematical argumentation as a pedagogical approach. At an early stage, IBA proposes students' involvement in addressing a purposeful inquiry question, advancing evidence which is used to form a conclusion, and justifying the conclusion through epistemically acceptable reasoning in a (mathematical or non-mathematical) context. As students progress, then a deeper focus on audience, articulation of parameters, and critique of self and others could be addressed to deepen the quality of the argument focus.

Finally, many of the results obtained in this study are likely attributable to both implementation of IBA and a related shift towards a culture of Knowledge Building. The extent to which the principles of Knowledge Building align and are intertwined with elements of IBA makes distinguishing any potential impact of IBA impossible to separate with any certainty from any potential impact of the Knowledge Building focus.

Both the proposed model of Inquiry Based Argumentation and the Signature Elements were developed from Design-Based Research (Cobb et al., 2003; Lesh, 2002): the very nature of which is to develop theory. While this enabled a deeper focus on intervention and reflection, the nature of Design Research means that it is very specific to a context and while this offers specific opportunities, there are also inherent limitations. The final chapter will consider some of the implications of the research reported in this dissertation, consider some limitations to the study and identify areas which would benefit from further research.

9 Conclusion

The work is by nature pragmatic; it must have some use beyond a philosophical orientation. That is, the work aims to specify learning processes involved and result in the development of some practical application. The results are therefore humble and specific to the context. (Cobb et al., 2003)

9.1 Introduction

The aim of this exploratory research was to develop pedagogical theory of inquiry-based argumentation in mathematics through multiple iterations of reflective-prospective cycles of improvement. In particular, the following questions were addressed:

1. What are key features of an Inquiry-Based Argument (IBA) model as implemented in a primary (elementary) mathematics setting?

2. What Signature Elements of Inquiry-Based Argument can serve to guide children's mathematical argumentation?

In response to the first research question, a proposed model of IBA was provided in the discussion chapter which sought to describe three integrated aspects of IBA: Context Knowledge, Argumentation Knowledge, and Mathematical Knowledge. While each of these domains was described, it was not suggested that they stand alone but rather that they are intertwined almost inextricably. However, each domain has potential for being developed in its own right and each offers potential to support the others. By the very nature of 'Mathematical Argumentation', neither the Mathematical Knowledge domain, nor the Argumentation Knowledge domain is optional to the model. However, the domain of Context Knowledge may be excluded, or at the very least marginalised, if the context comes from within the field of mathematics.

In response to the second research question, Signature Elements were identified that essentially define the underlying principles of IBA: The necessity to incorporate a purposeful inquiry question, the need for evidence to form a conclusion, and the justifying of the conclusion through epistemically acceptable reasoning. While more advanced elements were also provided, these are the core elements that are required.

It was in the development of the model of Inquiry-Based Argumentation and of the Signature Elements that this research has value to mathematics education. Of necessity, this research is extremely limited: it can neither be replicated nor generalised to other contexts, and is not sufficiently advanced that it would be suggested to even consider scaling-up this approach for any form of broad implementation in schools. What it does provide is a starting point for further research. In the sections below, the potential that IBA has to address school-based needs is considered, along with some of the more specific limitations to the study. The primary purpose of the study was to begin to consider IBA as a pedagogical tool in mathematics and thus the final section of this chapter will consider a small fraction of the research that could be undertaken related to IBA.

9.2 Significance for Mathematics Education

In the introduction, Australia's declining position in mathematical literacy, relative to other countries, was discussed. When this decline is considered in conjunction with decreased participation rates in enabling mathematics subjects at the senior secondary and tertiary levels, it is apparent that Australia has much to be concerned about in terms of future workforce competency; in particular, the needs to meet the demands of STEM occupations.

The Australian National Numeracy Review Report (NNRP, 2008), came about in response to a need for improving numeracy and mathematics learning competencies within Australia (p. vii). This document was underpinned by research-based evidence sourced both domestically and internationally. The NNRP sought to address this research in light of Australian student performance in both national benchmarks and international testing results, such as PISA and TIMSS. The outcome of this report was a series of recommendations, directed specifically towards the Australian context. Four recommendations were made for improving numeracy outcomes. Two of these are not germane to the research described here as one addresses actual time spent teaching mathematics in schools and the other involves systemic testing. The remaining two are provided below:

1. That all systems and schools recognise that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom …

2. That from the earliest years, greater emphasis be given to providing students with frequent exposure to higher-level mathematical problems rather than routine procedural tasks, in contexts of relevance to them, with increased opportunities for students to discuss alternative solutions and explain their thinking. (2008, p.xii).

These recommendations have interesting parallels when considered in conjunction with the US Professional Standards for Teaching Mathematics (NCTM, 1991) which state that mathematics classrooms must:

- move towards becoming mathematical communities rather than collections of individuals;
- use logic and mathematical evidence for verification rather than the teacher being the sole authority;
- focus on reasoning rather than memorisation;
- use problem solving rather than mechanistic answer-finding; and,
- make connections between mathematics, its ideas, and its applications rather than being seen as a body of isolated concepts and procedures.

In both instances, there is a focus on problem solving, reasoning, and communication: although the NCTM takes communication a step further and identifies and specifies learning communities as distinct from merely increased communication through the inclusion of discussion and explanation. Another commonality is the focus on contextualised learning, or a focus on mathematics that enables connections to be made with and across the curriculum. This research study provides strong support for the potential of IBA to provide a means by which to address these goals in the classroom.

Problem Solving, Reasoning and Communication

The Signature Elements of IBA identified in this dissertation suggest that the opportunities that students had to explain their understandings, pose problems and pathways for possible solution, discuss representations and interpretations, and negotiate ideas, served to engage students in addressing problems. While students worked frequently in smaller groups, they were repeatedly given opportunities to come together as a class and share their group progress, receive feedback from the whole-class community and revise approaches with a final view to pooling knowledge to reach a whole-of-class response.

In doing so, students were afforded multiple opportunities to envisage, plan for, gather, represent, interpret and explain evidence: evidence in the form of mathematically-based findings. In this respect, the second of the NCTM Standards was addressed as students adopted an evidence-based focus while the teacher took the position of facilitator and guide. The teacher's role was to model argumentation practices, and to act as a class resource to be drawn upon as needed. There was still significant input and management from the teacher and this should be expected with younger students who were engaging with argumentation for the first time. However, there was a shifting over time from the teacher modelling questioning practices to the teacher becoming a less dominant member of the class. As students became familiar with processes associated with argumentation, they became more independent and it was sufficient that the teacher pose a small number of questions to provoke further thinking and alternate considerations to deepen or direct the mathematical focus. In terms of practical application in classrooms, this suggests that it is feasible to promote a community of practice approach and to do so within a reasonable time-frame; although a whole of school approach would of course be ideal in terms of practical application in classrooms. The implementation of IBA in a classroom which is striving towards becoming a Knowledge Building community has potential for addressing this first goal of becoming a community.

The NNRP recommendations and NCTM Standards place increased focus on problem-solving and reasoning, rather than memorisation (of facts and procedures) and mechanistic answer finding. This focus also ties with the intent of the new Australian Curriculum for Mathematics with the inclusion of Problem-Solving and Reasoning as specific proficiencies to be addressed in the classroom (ACARA, 2014a). The model of IBA used with students to facilitate the argumentation process in the classroom (Figure 7.9) underscores the essential role that the Problem or Question has: essentially IBA is Problem-Solving. In the process of addressing a problem, students are guided and expected to not only respond with mathematically based evidence, but to reason why their evidence leads to a claim and justify their reasoning and evidence as required. Reasoning is a core element of IBA. Students become problem solvers in a variety of contexts, and using and applying a range of mathematical knowledge and processes which they are required to explain and justify. In fact, the very nature of argumentation, as described here, demands that students move along Berland and Reiser's (2009) hierarchy from understanding to explanation and persuasion: thereby necessitating mathematical communication at increasingly demanding and complex levels. Finally, Reasoning itself is central to argumentation regardless of whether the CER model (McNeill & Krajcik, 2008; McNeill & Martin, 2011; Zembal-Saul et al., 2013) or Toulmin model (Toulmin, 1958; Toulmin et al., 1984) are adopted for use. However, the CER model makes the role of reasoning explicit and clearer to younger students than perhaps backings and warrants would.

The research reported here provides a model that may be used by teachers (or researchers) to implement a problem-solving approach in the classroom, while maintaining student focus on the important nature of evidence and reasoning in addressing mathematical problems. The argumentation component of the approach addresses the specifics of what is required in an argument and provides scaffolding to the teacher and students when engaging in, and reporting on conclusions in IBA. The very nature of IBA requires that students do negotiate pathways and communicate responses. IBA has the potential to deepen that through a specific focus on justification of responses and the mathematical nature of the evidence required.

Making connections

The other area addressed by both the NCTM and the NNRP was the need to make connections between mathematics and its applications rather than being seen as a body of isolated concepts and procedures. While IBA has significant potential to make connections within mathematics and between mathematics and other areas of the curriculum and 'real' life, this is not ensured by simply adopting IBA as a pedagogical approach. Ainley et al. (2006) discuss the role of purpose and utility in task design in order to enable students to see the value of mathematics. IBA offers opportunity (partly through question design and partly through the teacher's envisaging of potential pathways to address the question) to demonstrate purpose and utility to a high degree, and in this way address mathematical connections, ideas and applications. IBA offers affordances for a students' valuing of mathematics if the principles of purpose and utility are adhered to. On this note, it would also appear that mathematical problems that are contextualised within mathematics have a different function than those contextualised outside the field: that these 'types' of contexts have the potential to inculcate students into the practices of pure mathematics and applied mathematics as separate branches of the same discipline. The author would suggest that both of these contexts are valuable but serve different purposes in developing mathematical and argumentation knowledge but certainly more research is needed in this area.

9.3 Limitations

The purpose of the research addressed in this dissertation was to begin to explore possibilities for the use of Argumentation in mathematics teaching and learning at the primary level of schooling. As such, Design-Based Research (Cobb et al., 2003) was selected due to the interventionist nature. While studies such as this one are not statistically generalizable, it is important that the value of such studies is recognised in terms of transferability to new settings:

> More than any other form of research, design experiments have a real potential in bringing about change in teaching and learning. As they are both rigorous and flexible, the decisions made for changing the curriculum and for documenting the teaching and learning that occurs optimize both learning and the research design in an ongoing and cumulative way. In this second sense, the results of design experiments are generalizable, or rather transferable, to new settings, which they inform and influence, and which in turn generate new knowledge that has the potential to further change what and how we teach. (Roth, 2005, p. 82)

The class described, and the teacher, were experienced in the process of Inquiry-Based Learning and as such, had already proceeded some way to developing a classroom culture that was conducive to Inquiry and thus Inquiry-Based Argumentation. As such, the class was unusual and it is not suggested that another class, particularly one not familiar with this particular model of IBL, would respond in similar ways or develop similar understandings. Further, this class was aware that they were part of a research project and were co-opted as "joint researchers", thus they felt they had a vested interest in the development of models for IBA and potentially engaged at a metacognitive level that would be atypical under other circumstances.

Rather than attempting to provide a model of IBA that can be taken into a classroom and adopted, this research aimed to create some initial frameworks for the further study and research of IBA in mathematics. The tangible results are the classroom models for guiding students through IBL and IBA, a rubric for assessing the strength of argument both as a process and as a product as it might apply to mathematics and mathematical reasoning, and a model that attempts to illustrate possible components of IBA – Mathematical Knowledge, Argumentation Knowledge and Context Knowledge. Each of these tangibles is intended to be a tool for use in further research, both by the author and hopefully other researchers. Each tool is intended to be critiqued, improved, challenged and strengthened: as would be expected of a Knowledge Building research community. In the next section, a few key avenues of research are identified that might seek to address the limitations of this study, extend the work done here, and to address the wealth of work that could not be covered here due to constraints of time.

9.4 Suggestions for Further Research

One of the primary goals of Design-Based Research (Cobb et al., 2003) is to take an iterative approach with successive cycles of generating conjecture, developing ideas, testing them, and reflecting critically on the progress and process and then generating further ideas and theories. Thus research in this dissertation addresses one class of students over a relatively brief period of time. There is certainly a need for the research here to be undertaken with a broader range of ages and ideally as a longitudinal study. Opportunities to identify what IBA would 'look like' in junior primary and early childhood settings would give valuable insight into the possibilities for argumentation in the earlier years. A longitudinal study might offer insight into students' potential for developing IBA if engaged with such over many years. Studies leading to a greater understanding of IBA in terms of student age, longitudinal development, school and class culture, might also provide further opportunities to refine the research tools provided in this dissertation, acting as a service to research itself.

Each specific aspect of the Argumentation models proposed would benefit from additional research also. The role that representations play in organising and making sense of evidence is important and a site of increased mathematical understanding and appreciation. The nature of changes to representations across the course of single or multiple Inquiries would also be a worthy topic of research. Another potential area for research that may impact upon the nature of the model is the role of rebuttal. While not addressed in the research here, and upon reflection, there would appear to be potential for having students identify their own alternate conceptions and use a mechanism of rebuttal to reason why these conceptions are not more suitable forms of evidence than those they provide in conclusion.

Student confidence was not specifically addressed in this research. Despite this, it is possible to conjecture that students working towards a common goal of Knowledge Building would feel less anxious and more likely to risk-take than when engaged in individual right-wrong approaches to mathematics. The opportunities to collaborate, use preferred learning styles, and to ultimately arrive at a defensible solution would be thought likely to build confidence. This is an area where more research would be ideal as the limited research undertaken thus far into Inquiry-Based Learning suggests improved affect; however, the study in question was limited in nature (Fielding-Wells & Makar, 2008a).

Finally, another question regarding an individual aspect of the students' arguments would be the influence of audience, including, representations, interpretations and level of detail provided in the claim and reasoning. For example, one important aspect that could be addressed in further research is the *tenor* of the audience – if the students are preparing arguments for professional mathematicians, we would seek different forms of evidence and representations of that evidence than if they were sharing their findings with the class below them. Another may be the *mode* of the argument: do students alter their argument when presenting orally to a live audience that can challenge in the moment, or in written format, where they are presenting to an audience that is unseen and unable to immediately challenge?

On a final note, the issues surrounding audience are highly significant and worthy of considerably more research. We have seen the importance of context, and the audience serves to add to that context. In many circumstances, the nature of the audience would serve to influence the evidence provided by the students and the selection and representation of their evidence. Two specific factors would need to be addressed: one is that the students cannot help but be aware when they are presenting an argument that everyone in their classroom is aware of the question, the parameters, and the methodology used. Thus, even though they are asked to explain their findings, the students in this study tended to gloss over these aspects. However, an audience external to the classroom would necessitate an explanation of these factors. Secondly, the audience will influence field (the perspective from which the context is taken and the language used), tenor (the relationship between the interlocutor and the audience), and mode (written, spoken). For example, a student creating an authentic formal proposal for the costing, design, and ultimate recommendation of a new playground to the Parent’s and Citizen’s Committee will look and sound far different from the nature of the in-class arguments seen in the Pyramid and Barbie units which were maintained in class. Experiences at considering audience would surely be of benefit to students in constructing arguments. At the very least there is scope for significant research here.

9.5 **A Final Note**

The work as presented here is by nature limited and specific to a context. However, that does not necessarily diminish the significance. Research in science education has identified many benefits to the use of argumentation as a pedagogical approach in science teaching and learning. Thus the intent of the research described here was to explore whether similar approaches could be used in primary mathematics: if so, what this might ‘look’ like in the classroom? What might be the key features? The Signature Elements? How might students shift to such practices?

There was extensive possibility in terms of analysis of the data collected. It was tempting to make closer comparisons to science, to enumerate the benefits to students, or to even focus extensively on the mathematical development of the students. However, parameters had to be set to this inquiry, and a decision was made to restrict the dissertation to addressing more theoretical aspects in order to establish a hook on which to hang future research.

So, there is now something of an articulated idea of what IBA might ‘look’ like. A model on which to focus planning of further research has been tentatively proposed, and Signature Elements suggested. This research is not presented as a final answer to problems of developing deep understandings in students, or engaging them more deeply in mathematics, rather it is a humble and naïve starting point with some exciting possibilities.

As such it is offered up to the research communities of Mathematics Education and the Learning Sciences with the dearest wish that it provokes response.

Appendices

Appendix A: Barbie Unit Curriculum Connections

<table>
<tr><td colspan="2">Mathematics</td></tr>
<tr><td>Ways of Working Year 5</td><td>Content</td></tr>
<tr><td>Students are able to:
• pose questions and make predictions based on experience in similar situations
• plan activities and investigations to explore concepts, pathways and strategies and solve mathematical questions, issues and problems
• identify and use mental and written computations, estimations, representations and technologies to generate solutions and check for reasonableness of solutions
• evaluate their own thinking and reasoning, in relation to the application of mathematical ideas, strategies and procedures
• communicate and justify thinking and reasoning, using everyday and mathematical language, concrete materials, visual representations and technologies</td><td><u>Number – Year 5</u>

• Common and mixed fractions involving denominators to tenths can be represented as a collection of objects, on number lines and as parts of measure to solve problems

<u>Number – Year 7</u>

• Whole numbers, including positive and negative numbers, and common and decimal fractions can be ordered and compared using a number line.
• Common fractions can be represented as equivalent fractions, decimals and percentages for different purposes
• Percentages, rate, ratio and proportion can be used to describe relationships between quantities and to solve problems in practical situations involving money, time and other measures

<u>Chance and Data – Year 5</u>

• Data collected from experiments or observations can be organised in tables and graphs and used to respond to questions about the likelihood of possible outcomes of events
• Collected data can be used to justify statements and predictions
• Sets of data may contain expected or unexpected variation, and this may mean that additional data are needed

<u>Chance and Data – Year 7</u>

• Sample data drawn from a given population can be summarised, compared and represented in a</td></tr>
</table>

	variety of ways • Variation and possible causes of bias can be identified in data collections.
Health and Physical Education	
Ways of Working	**Content**
	Personal Development • Representations of people, including stereotypes, influence the beliefs and attitudes that people develop about themselves and others
The Arts	
Ways of Working	**Content**
Students are able to: • select and develop ideas for arts works, considering different audiences and different purposes, using arts elements and languages • create and shape arts works by organising arts elements to express personal and community values, beliefs and observations • respond to arts works by identifying and interpreting the influences of social, cultural and historical contexts, using arts elements and languages	Visual Arts • Colour shades (adding black to a colour) and tints (adding colour to white) are used to create balance, contrast and patterns • Continuous, broken and hatched lines are used to create balance, contrast, space and patterns • Curved, angular, symmetrical, asymmetrical and overlapping shapes are used to create balance, contrast and patterns • Texture creates contrast and patterns using lines, rubbings and markings
English	
Ways of working	**Content**
	Speaking and Listening • The purpose of speaking and listening includes informing, presenting simple arguments, negotiating relationships and transactions, and seeking opinions of others • In presentations, speakers make meaning clear through the selection and sequencing of ideas and information and the use of visual aids as support

Appendix B: Pyramid Unit Curriculum Connections

<table>
<tr><th colspan="2">Mathematics</th></tr>
<tr><th>Ways of Working</th><th>Content</th></tr>
<tr><td>Students are able to:

• identify and describe the mathematical concepts, strategies and procedures required to generate solutions

• pose questions and make predictions based on experience in similar situations

• plan activities and investigations to explore concepts, pathways and strategies and solve mathematical questions, issues and problems

• identify and use mental and written computations, estimations, representations and technologies to generate solutions and check for reasonableness of solutions

• make statements, predictions, inferences and decisions based on mathematical interpretations

• evaluate their own thinking and reasoning, in relation to the application of mathematical ideas, strategies and procedures

• communicate and justify thinking and reasoning, using everyday and mathematical language, concrete materials, visual representations and technologies

• reflect on mathematics and identify the contribution of mathematics</td><td><u>Space – Year 5</u>

• Geometric features, including parallel and perpendicular lines, acute, right, obtuse and reflex angles, and vertex, edge and base, can be used to sort shapes and objects into broad family groups

• Defining features, including edges, angle sizes and parallel lines, are used to make accurate representations of 2D shapes and 3D objects.

• 3D objects can be visualised or constructed using nets

<u>Space – Year 7</u>

• Geometric conventions, including length, angle size and relationships between faces, are used to classify 2D shapes and 3D objects, including part and composite shapes
e.g. isosceles triangles have two equal sides and two equal base angles.

• 3D objects can be constructed from plans, nets and isometric diagrams

<u>Measurement – Year 5</u>

• Standard units, including (examples provided) and a range of instruments are used to measure and order attributes of objects, including length, area, volume, mass, time, and angles</td></tr>
<tr><th colspan="2">English</th></tr>
<tr><th>Ways of working</th><th>Content</th></tr>
<tr><td>• identify the relationship between audience, purpose and text type</td><td><u>Speaking and Listening – Year 5</u></td></tr>
</table>

	• The purpose of speaking and listening includes informing, presenting simple arguments, negotiating relationships and transactions, and seeking opinions of others • In presentations, speakers make meaning clear through the selection and sequencing of ideas and information and the use of visual aids as support <u>Speaking and Listening – Year 7</u> • The purpose of speaking and listening includes advancing opinions, discussing, persuading others to a point of view, influencing transactions, and establishing and maintaining relationships *e.g. debating or discussing a current topic from a particular viewpoint can persuade others.* • Active listeners identify ideas and issues from others' viewpoints and clarify meanings to justify opinions and reasoning.

Appendix C: Coding (Sample)

Guiding principles/signature elements			
Purpose	Ill-structured		PI-
	Ambiguous		PA-
	Rich/Complex - pathways		PP-
	Contextualises the inquiry		PC-
	Connections	Real life	PCL
		Across curriculum	PCC
		Across mathematics	PCM
	Provides a need/focus for question		PF-
	Engage students	Compelling	PEC
	Engage students	Driver	PED
	Engage students	Curiosity	PEU
	Engage students	Provocative	PEP
	Authentic Audience		PAA
Question	Specific		QS-
	Researchable		QR-
	Contentious/Unknown		QC-
	Mathematical		QM-
	Represents the intent of the purpose		QR-
Advance Evidence	Responds to Question		AQ-
	Alignment (on right track but not exact)		AA-
	Reliant on maths		AR-
	Mathematically acceptable		AM-
Conclusion	Forms and justifies a conclusion	Claim	CJC
	Forms and justifies a conclusion	Evidence	CJE
	Forms and justifies a conclusion	Reasoning	CJR
	Responds to the inquiry question		CI-
	Based on mathematical evidence		CM-
	Reflects the context		CC-
Defends and negotiates conclusions	Defends CER		DC-
	Defends methodology		DM-
	Critically examines the CER of others (including the methodology and mathematics)		DCO
	Reflects on criticism of others	Peers	DCP

	Reflects on criticism of others	Teacher	DCT
	Collaborative to draw best conclusion		DC-
Parameters	Recognises Limitations of evidence		PL-
	Qualifiers		PQ-
	Rebuttals		PR-
Consideration of Audience	Matches CER to intended audience	Consideration of representations	AAR
		Consideration of amount and type of data to present	AAD
Practices and tools which encourage and support			
Evidence Cycle	Diagram		ED-
	Evidentiary Focus	Part of class culture	EEC
		Focussed students	EEF
		Helped with links – Q & C	EEL
		Set the stage for argument	EES
	Classroom Culture	Collaborative nature	ECC
		Acceptance of multiple perspectives	ECP
		Reflecting on contributions	ECR
		Decentration	ECD
		Accountability	ECA
	Difficulties	Establishing a referential perspective	EDP
		Scaffolding	EDS
	Managing the inquiry	Maintaining maths focus	EMF
		Maintaining momentum	EMM
	Task Knowledge	Context knowledge	ETC
		Argumentation knowledge	ETA
		Mathematical knowledge	ETM
	Classroom discourse	Vocab of maths	EDM
		Vocab of argument	EDA
		Vocab of context	EDC
	Explicit teaching of argument structure		ES-
	Explicit teaching of argument quality		EQ-

Context, Mathematics, Argument			
Interactions	Scaffolding	Maths scaffold argument	MSA
		Argument scaffold maths	ASM
		Argument scaffold context	ASC
		Context scaffold argument	CSA
		Maths scaffold context	MSC
		Context scaffold maths	CSM
Context	Inductive vs Deductive reasoning		IDR
	Evidence – Quality vs Quantity – dominant ‘type’		EQQ
	Connections	Real life	CCR
		Across curriculum	CCC
		Across mathematics	CCM
	Field in argument structure		CFA

References

Aberdein, A. (2009). Mathematics and argumentation. *Foundations of Science, 14*(1), 1-8. doi: 10.1007/s10699-008-9158-3

ACARA. (2012). *Shape of the Australian curriculum*. Retrieved from http://www.acara.edu.au/verve/_resources/The_Shape_of_the_Australian_Curriculum_v4.pdf.

ACARA. (2014a). *Australian curriculum: Mathematics v7.0*. Retrieved from http://www.australiancurriculum.edu.au/.

ACARA. (2014b). *Australian curriculum: Science v7.0*. Retrieved from http://www.australiancurriculum.edu.au/.

Ainley, J., Pratt, D., & Hansen, A. (2006). Connecting engagement and focus in pedagogic task design. *British Educational Research Journal, 32*(1), 23-38.

Allmond, S., & Makar, K. (2010). Developing primary students' ability to pose questions in statistical investigations. In C. Reading (Ed.), Data and context in statistics education: Towards an evidence-based society. *Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia.* Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

Allmond, S., Wells, J., & Makar, K. (2010). *Thinking through mathematics: Engaging students with inquiry-based learning*. Carlton South, VIC: Education Services Australia.

Anderson, R.D. (2002). Reforming science teaching: What research says about inquiry. *Journal of Science Teacher Education, 13*(1), 1-12.

Aristotle. (1954). *The rhetoric and the poetics of Aristotle* W. R. Roberts, Trans.). In F. Solmsen (Ed.), *Aristotle: Rhetoric and poetics*. New York, NY: Modern Library. (Original work published 250 BCE).

Asterhan, C.S.C., & Schwarz, B.B. (2007). The effects of monological and dialogical argumentation on concept learning in evolutionary theory. *Journal of Educational Psychology, 99*(3), 626-639.

Astington, J.W. (1994). *The child's discovery of mind*. London: Fontana Press.

Atweh, B., Miller, D., & Thornton, S. (2012). The Australian curriculum: Mathematics - world class or deja vu? In B. Atweh, M. Goos, R. Jorgensen & D. Siemon (Eds.), *Engaging the Australian National Curriculum: Mathematics - Perspectives from the Field. O*nline publication: Mathematics Education Research Group of Australasia pp. 1-18.

Australian Academy of Science. (2006). *Mathematics and statistics: Critical skills for Australia's future*. Retrieved from http://www.review.ms.unimelb.edu.au

Barrington, F. (2011). *2010 Year 12 mathematics student numbers*. Melbourne: Australian Mathematical Sciences Institute (AMSI).

Barrington, F. (2012). *2011 Year 12 mathematics student numbers*. Melbourne: Australian Mathematical Sciences Institute (AMSI).

Barrington, F. (2013). *2012 Year 12 mathematics student numbers*. Melbourne: Australian Mathematical Sciences Institute (AMSI).

Bell, P., & Linn, M.C. (2000). Scientific arguments as learning artifacts: Designing from learning from the web with KIE. *International Journal of Science Education, 22*(8), 797-817.

Bereiter, C. (1994). Implications of postmodernism for science, or, science as progressive discourse. *Educational Psychologist, 29*(1), 3-12. doi: 10.1207/s15326985ep2901_1

Bereiter, C., & Scardamalia, M. (1996). Rethinking learning. In D. R. Olson & N. Torrance (Eds.), *The handbook of education and human development* (pp. 485-513). Malden, MA: Blackwell Publishers.

Bereiter, C., Scardamalia, M., Cassells, C., & Hewitt, J. (1997). Postmodernism, knowledge building, and elementary science. *The Elementary School Journal, 97*(4), 329-340. doi: 10.2307/1002350

Berland, L.K., & Forte, A. (2010). *When students speak, who listens? Constructing arguments in classroom argumentation.* Paper presented at the Proceedings of the 9th International Conference of the Learning Sciences - Volume 1, Chicago, IL.

Berland, L.K., & Reiser, B.J. (2009). Making sense of argumentation and explanation. *Science Education, 93*(1), 26-55. doi: 10.1002/sce.20286

Bermejo-Luque, L. (2006). Toulmin's model of argument and the question of relativism. In D. Hitchcock & B. Verheij (Eds.), *Arguing on the Toulmin model: New essays in argument analysis and evaluation.* (pp. 71-85). Netherlands: Springer

Biro, J., & Siegel, H. (1992). Normativity, argumentation and an epistemic theory of fallacies. In F. H. van Eemeren, R. Grootendorst, J. A. Blair & C. A. Willard (Eds.), *Argumentation illuminated* (pp. 85-103). Amsterdam: International Centre for the Study of Argumentation.

Blair, J.A. (2012). Argumentation as rational persuasion. *Argumentation, 26*(1), 71-81. doi: 10.1007/s10503-011-9235-6

Boulton-Lewis, G.M. (1998). Children's strategy use and interpretations of mathematical representations. *The Journal of Mathematical Behavior, 17*(2), 219-237. doi: http://dx.doi.org/10.1016/S0364-0213(99)80060-3

Bransford, J.D., Brown, A., & Cocking, R.R. (Eds.). (2000). *How people learn: Brain, mind, experience, and school: Expanded edition*. Washington, D.C.: National Academy Press.

Brown, A.L., & Campione, J.C. (1990). Communities of learning and thinking, or a context by any other name. *Human Development, 21*, 108-125.

Brown, A.L., & Campione, J.C. (1996). Psychological theory and the design of innovative learning environments: On procedures, principles and systems. In L. Schauble & R. Glaser (Eds.), *Innovations in learning: New environments for education* (pp. 289-325). Mahwah, NJ: Lawrence Erlbaum

Brown, H. (2004). Walking into the unknown: Inquiry-based learning transforms the English classroom. *The English Journal, 92*(2), 43 - 48.

Brown, J.S., Collins, A., & Duguid, P. (1989). Situated cognition and the culture of learning. *Educational Researcher, 18*(1), 32-42.

Brown, R. (2007). Exploring the social positions that students construct within a classroom community of practice. *International Journal of Educational Research, 46*(3–4), 116-128. doi: http://dx.doi.org/10.1016/j.ijer.2007.09.012

Carew, D. (2004). Educating the 21st century citizen. *The Catalyst, 33*(10), 3-9.

Chin, C., & Osborne, J. (2010). Supporting argumentation through students' questions: Case studies in science classrooms. *Journal of the Learning Sciences, 19*(2), 230 - 284.

Chu, S. (2008). Grade 4 students’ development of research skills through inquiry-based learning projects. *School Libraries Worldwide, 14*(1), 10-38.

Chuy, M., Zhang, J., Resendes, M., Scardamalia, M., & Bereiter, C. (2011). *Does contributing to a knowledge building dialogue lead to individual advancement of knowledge?* Paper presented at the CSCL2011 Computer Supported Collaborative Learning: Connecting Research to Policy and Practice, Hong Kong, China.

Cifarelli, V.V. (1998). The development of mental representations as a problem solving activity. *The Journal of Mathematical Behavior, 17*(2), 239-264. doi: http://dx.doi.org/10.1016/S0364-0213(99)80061-5

Clarke, A.E. (2005). *Situational analysis: Grounded theory after the postmodern turn*. Thousand Oaks, CA: Sage Publications.

Clements, D.H., & Battista, M.T. (1990). Research into practice *The Arithmetic Teacher, 38*(1), 34-35.

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Cobb, P., Wood, T., & Yackel, E. (1993). Discourse, mathematical thinking and classroom practice. In E. A. Formena, N. Minick & C. A. Stone (Eds.), *Contexts for learning: Sociocultural dynamics in children's development*. New York, NY: Oxford University Press.

Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist, 31*(3-4), 175-190. doi: 10.1080/00461520.1996.9653265

Collins, A. (2006). Cognitive apprenticeship. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 47-60). Cambridge, MA: Cambridge University Press.

Collins, A., Brown, J.S., & Newman, S.E. (1989). Cognitive apprenticeship: Teaching the crafts of reading, writing and mathematics. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 453-494). Hillsdale, NJ: Lawrence Erlbaum Associates.

Confrey, J. (1985). *A constructivist view of mathematics instruction: A theoretical perspective*. Paper presented at the American Education Research Association, Chicago.

Confrey, J. (1990). Chapter 8: What constructivism implies for teaching. *Journal for Research in Mathematics Education. Monograph, 4*, 107-210. doi: 10.2307/749916

Conner, A. (2007). *Student teachers' conceptions of proof and facilitation of argumentation in secondary mathematics classrooms.* (Ph.D. dissertation), The Pennsylvania State University, Pennsylvania. Dissertations & Theses: Full Text. database.

Corbin, J.M., & Strauss, A. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory*. Thousand Oaks, CA: Sage Publications.

Crawford, B.A., Krajcik, J.S., & Marx, R.W. (1999). Elements of a community of learners in a middle schoolscience classroom. *Science Education, 83*(6), 701-723.

Dixon, J.K., Egendoerfer, L.A., & Clements, T. (2009). Do they really need to raise their hands? Challenging a traditional social norm in a second grade mathematics classroom. *Teaching and Teacher Education, 25*(8), 1067-1076. doi: http://dx.doi.org/10.1016/j.tate.2009.04.011

Driver, R., Asoko, H., Leach, J., Mortimer, E., & Scott, P. (1994). Constructing scientific knowledge in the classroom. *Educational Researcher, 23*(7), 5-12. doi: 10.2307/1176933

Driver, R., Newton, P., & Osborne, J. (2000). Establishing the norms of scientific argumentation in classrooms. *Science Education, 84*(3), 287-312. doi: 10.1002/(sici)1098-237x(200005)84:3<287::aid-sce1>3.0.co;2-a

Duschl, R.A. (2007). Quality argumentation and epistemic criteria. In S. Erduran & M. P. Jimenez-Aleixandre (Eds.), *Argumentation in science*. New York, NY: Springer.

Duschl, R.A., & Osborne, J. (2002). Supporting and promoting argumentation discourse in science education. *Studies in Science Education, 38*(1), 39-73.

Eraut, M. (1994). *Developing professional knowledge and competence*. New York, NY: Routledge.

Erduran, S. (2007). Methodological foundations in the study of argumentation in science classrooms. In S. Erduran & M. P. Jiménez-Aleixandre (Eds.), *Argumentation in science education* (Vol. 35, pp. 47-69). Voorburg, The Netherlands: Springer

Erduran, S., Simon, S., & Osborne, J. (2004). Tapping into argumentation: Developments in the application of toulmin's argument pattern for studying science discourse. *Science Education, 88*(6), 915-933. doi: 10.1002/sce.20012

Fielding-Wells, J. (2010). Linking problems, conclusions and evidence: Primary students' early experiences of planning statistical investigations. In C. Reading (Ed.), Data and context in statistics education: Towards an evidence-based society. *Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia.* Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

Fielding-Wells, J. (2014). Where's your evidence? Challenging young students' equiprobability bias through argumentation. In K. Makar, B. de Sousa, & R. Gould (Eds.), *Sustainability in statistics education. Proceedings of the Ninth International Conference on Teaching Statistics (ICOTS9, July, 2014), Flagstaff, Arizona*,. Voorburg,The Netherlands: International Statistical Institute.

Fielding-Wells, J., Dole, S., & Makar, K. (2014). Inquiry pedagogy to promote emerging proportional reasoning in primary students. *Mathematics Education Research Journal, 26*(1), 1-31.

Fielding-Wells, J., & Makar, K. (2008a). *Student (dis)engagement with mathematics*. Paper presented at the Australian Association of Research in Education (AARE), Brisbane, Australia. Retrieved from http://www.aare.edu.au/08pap/mak08723.pdf

Fielding-Wells, J., & Makar, K. (2008b). *Using mathematical inquiry to engage student learning within the overall curriculum* In J. Adler & D. Ball (Eds.), Eleventh International Congress of Mathematics Education (ICME11) Mathematical Knowledge for Teaching (pp. 1-17). Monterrey, Mexico, 6-13 July 2008.

Fielding-Wells, J., & Makar, K. (2012). Developing primary students' argumentation skills in inquiry-based mathematics classrooms. In K. T. Jan van Aalst, Michael J. Jacobson and Peter Reimann (Ed.), *The future of learning: Proceedings of the 10th International Conference of the Learning Sciences [ICLS 2012] - volume 2 short papers, symposia, and abstracts.* (pp. 149-153). Sydney, NSW, Australia: International Society of the Learning Sciences.

Fielding-Wells, J., & Makar, K. (2013). *Inferring to a model: Using inquiry-based argumentation to challenge young children's expectations of equally likely outcomes*. Paper presented at the The Ninth International Conference on Statistical Reasoning, Thinking and Literacy Superior Shores, Minnesota

Flick, U. (2009). *An introduction to qualitative research* (4 ed.). London, UK: SAGE.

Forman, E.A., Larreamendy-Joerns, J., Stein, M.K., & Brown, C.A. (1998). "You're going to want to find out which and prove it": Collective argumentation in a mathematics classroom. *Learning and Instruction, 8*(6), 527-548. doi: 10.1016/s0959-4752(98)00033-4

Giannakoulias, E., Mastorides, E., Potari, D., & Zachariades, T. (2010). Studying teachers' mathematical argumentation in the context of refuting students' invalid claims. *The Journal of Mathematical Behavior, 29*(3), 160-168. doi: 10.1016/j.jmathb.2010.07.001

Golder, C., & Coirier, P. (1994). Argumentative text writing: Developmental trends. *Discourse Processes, 18*(2), 187-210.

Goos, M. (2004). Learning mathematics in a classroom community of inquiry. *Journal for Research in Mathematics Education, 35*(4), 258-291.

Greeno, J.G., & Hall, R.P. (1997). Practicing representation: Learning with and about representational forms. *The Phi Delta Kappan, 78*(5), 361-367. doi: 10.2307/20405797

Harel, G. (2008). DNR perspectives on mathematics curriculum and instruction, Part I: Focus on proving. *ZDM: The International Journal on Mathematics Education, 40*(3), 487-500. doi: 10.1007/s11858-008-0104-1

Harel, G., & Koichu, B. (2010). An operational definition of learning. *The Journal of Mathematical Behavior, 29*(3), 115-124. doi: 10.1016/j.jmathb.2010.06.002

Howe, C., & Mercer, N. (2007). *Children's social development, peer interaction and classroom learning (Primary Review Research Survey 2/1b)*. Oxon, UK : Routledge.

Jiménez-Aleixandre, M.P., Bugallo Rodríguez, A., & Duschl, R.A. (2000). "Doing the lesson" or "doing science": Argument in high school genetics. *Science Education, 84*(6), 757-792.

Jimenez-Aleixandre, M.P., & Erduran, S. (2007). Argumentation in science education. In S. Erduran & M. P. Jimenez-Aleixandre (Eds.), *Argumentation in Science Education: An overview* (pp. 3 - 27). Voorburg, The Netherlands: Springer.

Kabiri, M.S., & Smith, N.L. (2003). Turning traditional textbook problems into open-ended problems. *Mathematics Teaching in the Middle School, 9*(3), 186-192.

Kelly, G.J., Druker, S., & Chen, C. (1998). Students' reasoning about electricity: Combining performance assessments with argumentation analysis. *International Journal of Science Education, 20*(7), 849-871. doi: 10.1080/0950069980200707

Kincheloe, J.L., & Tobin, K. (2009). The much exaggerated death of positivism. *Cultural Studies of Science Education, 4*(3), 513-528. doi: http://dx.doi.org/10.1007/s11422-009-9178-5

Kirschner, P.A., Sweller, J., & Clark, R.E. (2006). Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching. *Educational Psychologist, 41*(2), 75-86. doi: 10.1207/s15326985ep4102_1

Kneupper, C.W. (1978). Teaching argument: An introduction to the Toulmin model. *College Composition and Communication, 29*(3), 237-241.

Krajcik, J., Blumenfeld, P.C., Marx, R.W., Bass, K.M., Fredricks, J., & Soloway, E. (1998). Inquiry in project-based science classrooms: Initial attempts by middle school students. *The Journal of the Learning Sciences, 7*(3/4), 313-350. doi: 10.2307/1466790

Krummheuer, G. (1995). The ethnography of argumentation. In P. Cobb & H. Bauersfeld (Eds.), *Emergence of mathematical meaning* (pp. 229-269). Hillsdale, NJ: Lawrence Erlbaum Associates.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal, 27*(1), 29-63.

Lave, J. (1988). *Cognition in practice*. Boston, MA: Cambridge University Press.

Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge, MA: Cambridge University Press.

Lee, H.-J., & Herner-Patnode, L.M. (2007). Teaching mathematics vocabulary to diverse groups. *Intervention in School and Clinic, 43*(2), 121-126.

Leitão, S. (2000). The potential of argument in knowledge building. *Human Development, 43*(6), 332-360.

Lerman, S. (2001). Cultural, Discursive Psychology: A Sociocultural Approach to Studying the Teaching and Learning of Mathematics*. Educational Studies in Mathematics, 4*6(1-3), 87-113. doi: 10.1023/a:1014031004832

Lesh, R. (2002). Research design in mathematics education: Focusing on design experiments. In L. English (Ed.), Handbook of international research in mathematics education (pp. 27 - 49). Mahwah, NJ: Lawrence Erlbaum.

Lumer, C. (2010). Pragma-dialectics and the function of argumentation. *Argumentation, 24*(1), 41-69. doi: 10.1007/s10503-008-9118-7

Makar, K. (2010). Teaching primary teachers to teach statistical investigations: The uniqueness of initial experiences. In C. Reading (Ed.), Data and context in statistics education: Towards an evidence-based society. *Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8, July, 2010), Ljubljana, Slovenia.* Voorburg, The Netherlands: International Statistical Institute. www.stat.auckland.ac.nz/~iase/publications.php

Makar, K. (2011). Learning over time: Pedagogical change in teaching mathematical inquiry. In J. Clark, B. Kissane, J. Mousley, T. Spencer, & S. Thornton (Eds.), Mathematics: Traditions and [new] practices. *Proceedings of the 34th Annual Conference of the Mathematics Education Research Group of Australasia and the Australian Association of Mathematics Teachers*, Alice Springs, NT. Adelaide, SA: AAMT and MERGA.

Makar, K., & Fielding-Wells, J. (2011). Teaching teachers to teach statistical investigations. In C. Batanero, G. Burrill & C. Reading (Eds.), *Teaching statistics in school mathematics-challenges for teaching and teacher education* (Vol. 14, pp. 347-358): Voorburg, The Netherlands: Springer.

Makar, K., & Rubin, A. (2009). A framework for thinking about informal statisitical inference. *Statistics Education Research Journal, 8*(1), 82-105.

Marshall, H.H. (1996). Implications of differentiating and understanding constructivist approaches. *Educational Psychologist, 31*(3-4), 235-240. doi: 10.1080/00461520.1996.9653270

McInerney, D. (2006). *Developmental psychology for teachers: An applied approach*. Crows Nest, NSW: Allen & Unwin.

McNeill, K.L., & Krajcik, J. (2008). Inquiry and scientific explanations: Helping students use evidence and reasoning. In J. Luft, R. L. Bell & J. Gess-Newsome (Eds.), *Science as inquiry in the secondary setting* (pp. 121-134). Arlington, VA: National Science Teacher Association.

McNeill, K.L., & Krajcik, J. (2011). *Supporting grade 5-8 students in constructing explanations in science: The claim, evidence and reasoning framework for talk and writing*. Boston, MA: Pearson.

McNeill, K.L., & Krajcik, J. (2012). Middle school students' use of appropriate and inappropriate evidence in writing scientific explanations. In M. Lovett & P. Shah (Eds.), *Thinking with data* (pp. 233-266). New York, NY: Lawrence Erlbaum Associates.

McNeill, K.L., Lizotte, D.J., Krajcik, J., & Marx, R.W. (2006). Supporting students' construction of scientific explanations by fading scaffolds in instructional materials. *Journal of the Learning Sciences, 15*(2), 153-191. doi: 10.1207/s15327809jls1502_1

McNeill, K.L., & Martin, D.M. (2011). Claims, evidence, and reasoning. *Science and Children, 48*(8), 52-56.

McPhan, G., Morony, W., Pegg, J., Cooksey, R., & Lynch, T. (2008). *Maths? Why Not?* Canberra, ACT: DEEWR.

Minsky, M. (1961). Steps toward artificial intelligence. *Proceedings of the IRE, 49*(1), 8-30. doi: 10.1109/JRPROC.1961.287775

Muller Mirza, N., Perret-Clermont, A.-N., Tartas, V., & Iannaccone, A. (2009). Psychosocial processes in argumentation. In N. Muller Mirza & A.-N. Perret-Clermont (Eds.), *Argumentation and education: Theoretical foundations and practices* (pp. 67-90). New York: Springer.

Mullis, I.V.S., Martin, M.O., Foy, P., & Arora, A. (2012). *TIMSS 2011 international results in mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Centre.

National Numeracy Review Panel. (2008). *National numeracy review report*. Canberra, ACT: Author.

NCTM. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.

NCTM. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics. Retrieved from http://www.nctm.org/standards/content.aspx?id=4294967312.

Nettel, A.L., & Roque, G. (2012). Introduction. *Argumentation, 26*(1), 1-17. doi: 10.1007/s10503-011-9238-3

O'Keefe, D.J. (1977). Two concepts of argument. *Journal of the American Forensic Association, 13*(3), 121-128.

O'Keefe, D.J. (2012). Conviction, persuasion, and argumentation: Untangling the ends and means of influence. *Argumentation, 26*(1), 19-32. doi: 10.1007/s10503-011-9242-7

Organisation for Economic Co-operation and Development (OECD). (2006). *OECD Science, Technology and Industry Outlook*. Paris, France: OECD

Organisation for Economic Co-operation and Development (OECD). (2013). *Education policy outlook highlights: Australia*. Paris, France: OECD.

Office of the Chief Scientist. (2013). *Science, technology, engineering and mathematics in the national interest: A strategic approach*. Canberra, ACT: Australian Government.

Orsolini, M., & Pontecorvo, C. (1992). Children's talk in classroom discussions. *Cognition and Instruction, 9*(2), 113-136. doi: 10.1207/s1532690xci0902_2

Osborne, J., Erduran, S., & Simon, S. (2004). Enhancing the quality of argumentation in school science. *Journal of Research in Science Teaching, 41*(10), 994-1020. doi: 10.1002/tea.20035

Papert, S. (1993). *The children's machine: Rethinking school in the age of the computer*. New York, NY: Basic Books.

Pfannkuch, M. (2006, July). Informal inferential reasoning. In A. Rossman & B. Chance (Eds.), *Working cooperatively in statistics education: Proceedings of the Seventh International Conference on Teaching Statistics, Salvador, Bahia, Brazil*. Voorburg, The Netherlands: ISI

Piaget, J. (1960). *The child's conception of the world* (J. Tomlinson & A. Tomlinson, Trans. Vol. 1). Totowa, NJ: Littlefield, Adams & Co.

Pontecorvo, C., & Pirchio, S. (2000). A developmental view on children's arguing: The need of the other. *Human Development, 43*(6), 361-363.

Popper, K. (1972a). *Conjectures and refutations: The growth of scientific knowledge*. London, UK: Routledge.

Popper, K. (1972b). *Objective knowledge: An evolutionary approach* (Revised ed.). Oxford, UK: Clarendon Press.

Pratt, D., Johnston-Wilder, P., Ainley, J., & Mason, J. (2008). Local and global thinking in statistical influence. *Statistics Education Research Journal, 7*(2), 107-129.

Reitman, W.R. (1965). *Cognition and thought : An information processing approach*. New York, NY: Wiley.

Rogoff, B., Matusov, E., & White, C. (1996). Models of teaching and learning: Participation in a community of learners. In D. R. Olson & N. Torrance (Eds.), *The handbook of education and human development* (pp. 388-414). Malden, MA: Blackwell Publishers.

Roth, W-M, (2005). *Doing qualitative research: Praxis of method.* Rotterdam, The Netherlands: Sense Publishers.

Rowland, R.C. (1987). On defining argument evaluation. *Philosophy and Rhetoric, 20*(3), 140-159.

Rubenstein, R.N., & Thompson, D.R. (2002). Understanding and support children's mathematical vocabulary development. *Teaching Children Mathematics, 9*(2), 107-112.

Sampson, V., & Clark, D.B. (2008). Assessment of the ways students generate arguments in science education: Current perspectives and recommendations for future directions. *Science Education, 92*(3), 447-472. doi: 10.1002/sce.20276

Sampson, V., & Clarke, D. (2006, June). *Assessment of argument in science education: A critical review of the literature.* Paper presented at the 7th International Conference of the Learning Sciences. Bloomington Indiana.

Sanborn, J. (1992). The academic essay: A feminist view in student voices. In N. M. McCracken & B. C. Appleby (Eds.), *Gender issues in the teaching of English* (pp. 142-160). Portsmouth, NH: Boynton/Cook.

Sandoval, W.A., & Millwood, K.A. (2005). The quality of students' use of evidence in written scientific explanations. *Cognition and Instruction, 23*(1), 23 - 55.

Sandoval, W.A., & Millwood, K.A. (2007). What can argumentation tell us about epistemology? In S. Erduran & M. P. Jimenez-Aleixandre (Eds.), *Argumentation in science education* (pp. 71 - 88). Voorburg, The Netherlands: Springer.

Sawyer, R.K. (2006). Analyzing collaborative discourse. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 187-204). Cambridge: Cambridge University Press.

Scardamalia, M. (2002). Collective cognitive responsibility for the advancement of knowledge. In B. Smith (Ed.), *Liberal education in a knowledge society* (pp. 67-98). Chicago, IL: Open Court.

Scardamalia, M., & Bereiter, C. (2006). Knowledge building: Theory, pedagogy, and technology. In R. K. Sawyer (Ed.), *The Cambridge handbook of the learning sciences* (pp. 97-118). New York, NY: Cambridge University Press.

Scardamalia, M., & Bereiter, C. (2010). A brief history of knowledge building. *Canadian Journal of Learning and Technology, 36*(1), 1-16.

Scardamalia, M., & Bereiter, C. (2013). Fostering communities of learners and knowledge building: An interrupted dialogue. In J. C. Campione & K. Metz (Eds.), Children's learning in laboratory and classroom contexts: Essays in honor of Ann Brown (pp. 197-212). Hoboken, NY: Taylor and Francis.

Schoenfeld, A., H. (1991). On mathematics as sense-making: An informal attack on the unfortunate divorce of formal and informal mathematics. In J. F. Voss, D. N. Perkins & J. Segal (Eds.), *Informal reasoning and education* (pp. 311-344). Hillsdale, NJ: Erlbaum.

Schroeder, C. (1997). Knowledge and power, logic and rhetoric, and other reflections in the Toulminian mirror: A critical consideration of Stephen toulmin's contributions to composition. *JAC: A Journal of Composition Theory, 17*(1), 95-107.

Siegel, H., & Biro, J. (1997). Epistemic normativity, argumentation, and fallacies. *Argumentation, 11*(3), 277-292.

Silva, E. (2009). Measuring skills for 21st-century learning. *Phi Delta Kappan, 90*(9), 630-634.

Simon, H.A. (1973). The structure of ill-structured problems. *Artificial Intelligence, 4*(3-4), 181-201.

Simon, M.A. (1994). Learning mathematics and learning to teach: Learning cycles in mathematics teacher education. *Educational Studies in Mathematics, 26*(1), 71-94.

Simon, S., & Richardson, K. (2009). Argumentation in school science: Breaking the tradition of authoritative exposition through a pedagogy that promotes discussion and reasoning. *Argumentation, 23*(4), 469-493.

Skovsmose, O. (2008). Mathematical literacy and globalisation. In B. Atweh, A. Calabese Barton, M. C. Borba, N. Gough, C. Keitel, C. Vistro-Yu & R. Veithal (Eds.), *Internationalisation and globalisation in mathematics and science education*. Dordrecht, The Netherlands: Springer.

Stein, N.L., & Miller, C.A. (1993). A theory of argumentative understanding: Relationships among position preference, judgements of goodness, memory and reasoning. *Argumentation, 7*(2), 183-204.

Stygall, G. (1987). Toulmin and the ethics of argument fields: Teaching writing and argument. *Journal of Teaching Writing, 6*(1), 93-107.

Thomson, S., De Bortoli, L., & Buckley, S. (2013). *PISA 2012: How Australia measures up: The PISA 2012 assessment of students' mathematical, scientific and reading literacy*. Melbourne, Vic: Australian Council for Educational Research (ACER).

Thomson, S., De Bortoli, L., Nicholas, M., Hillman, K., & Buckley, S. (2011). *Challenges for Australian education: Results from PISA 2009: The PISA 2009 assessment of students' reading, mathematical, and scientific literacy.* Melbourne, Vic: Australian Council for Educational Research (ACER).

Thomson, S., Hillman, K., Wernert, N., Schmid, M., Buckley, S., & Munene, A. (2012a). *Highlights from TIMSS & PIRLS 2011 from Australia's perspective*. Melbourne, Vic: Australian Council for Educational Research (ACER).

Thomson, S., Hillman, K., Wernert, N., Schmid, M., Buckley, S., & Munene, A. (2012b). *Monitoring Australian Year 4 student achievement internationally: TIMSS and PIRLS 2011*. Melbourne: Australian Council for Educational Research (ACER).

Thomson, S., Hillman, K., Wernert, N., Schmid, M., Buckley, S., & Munene, A. (2012c). *Monitoring Australian Year 8 student achievement internationally: TIMSS and PIRLS 2011*. Melbourne: Australian Council for Educational Research (ACER).

Toulmin, S. (1958). *The uses of argument*. Cambridge, MA: Cambridge University Press.

Toulmin, S., Rieke, R., & Janik, A. (1984). *An introduction to reasoning* (2 ed.). New York, NY: Macmillan.

van Eemeren, F.H., & Grootendorst, R. (1992). *Argumentation, communication, and fallacies: A pragma-dialectical perspective*. Hillsdale, NJ: Lawrence Erlbaum Associates.

van Eemeren, F.H., & Grootendorst, R. (2004). *A systematic theory of argumentation: The pragma-dialectical approach*. New York, NY: Cambridge University Press.

van Eemeren, F.H., Grootendorst, R., Henkemans, F.S., Blair, J.A., Johnson , R.H., Krabbe, E.C.W., . . . Zarefsky, D. (1996). *Fundamentals of argumentation theory: A handbook of historical backgrounds and contemporary developments.* Mahwah, NJ: Lawrence Erlbaum.

von Glasersfeld, E. (1989). Constructivism in education. In T. Husen & T. N. Postlethwaite (Eds.), *The international encyclopedia of eduction, supplement volume 1* (pp. 162-163). Oxford, UK: Pergamon Press.

Voss, J.F., & Van Dyke, J. (2001). Argumentation in psychology: Background comments. *Discourse Processes, 32*(2-3), 89-111.

Wertsch, J.V. (1991). *Voices of the mind: A sociocultural approach to mediated action*. Cambridge, MA: Harvard University Press.

Wikipedia. (2013a). Barbie. Wikipedia. http://en.wikipedia.org/wiki/Barbie

Wikipedia. (2013b). Fulla (doll). Wikipedia. http://en.wikipedia.org/wiki/Fulla_%28doll%29

Wu, H. (1997). The mathematics education reform: Why you should be concerned and what you can do. *The American Mathematical Monthly, 104*(10), 946-954.

Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education, 27*(4), 458-477.

Zeidler, D.L. (1997). The central role of fallacious thinking in science education. *Science Education, 81*(4), 483-496.

Zembal-Saul, C., McNeill, K.L., & Hershberger, K. (2013). *What's your evidence? Engaging k-5 students in constructing explanations in science*. Boston, MA: Pearson.

Zhang, J., Hong, H.-Y., Scardamalia, M., Teo, C.L., & Morley, E.A. (2011). Sustaining knowledge building as a principle-based innovation at an elementary school. *Journal of the Learning Sciences, 20*(2), 262-307. doi: 10.1080/10508406.2011.528317

Zhang, J., Scardamalia, M., Reeve, R., & Messina, R. (2009). Designs for collective cognitive responsibility in knowledge-building communities. *Journal of the Learning Sciences, 18*(1), 7-44. doi: 10.1080/10508400802581676

Zohar, A., & Nemet, F. (2002). Fostering students' knowledge and argumentation skills through dilemmas in human genetics. *Journal of Research in Science Teaching, 39*(1), 35-62. doi: 10.1002/tea.10008